SOLUTIONS TO PROBLEM SET 16.4, page 733

4. The integral equals
\[
\frac{1}{i} \oint_{C} \frac{1 + 2(z + z^{-1})}{z(z + z_{1})(z - z_{2})} \, dz = \frac{1}{-4i} \oint_{C} \frac{2z^{2} + z + 2}{z(z - z_{1})(z - z_{2})} \, dz.
\]

where \(z_{1} = \frac{1}{4}\) (inside the unit circle) and \(z_{2} = 4\) (outside) give simple poles. The residue at \(z = 0\) is \(2/(z_{1}z_{2}) = 2\), and at \(z = \frac{1}{4}\) it is
\[
\frac{\frac{9}{16} + \frac{1}{4} + 2}{(\frac{1}{4})(\frac{1}{4} - 4)} = -\frac{38}{15}.
\]

This gives the answer
\[
\frac{2\pi i}{-4i} \left(2 - \frac{38}{15}\right) = \frac{4\pi}{15}.
\]

6. The integral equals
\[
\oint_{C} \frac{-\frac{1}{4}(z - 1/z)^{2}}{5z - 2(z^{2} + 1)} \, dz = \frac{-\frac{1}{4}}{-2i} \oint_{C} \frac{(z^{2} - 1)^{2}}{z^{2}(z^{2} - 5z/2 + 1)} \, dz.
\]

From (5*), Sec. 16.3, we obtain for the integrand of the last integral (without the factor in front of it) at \(z = 0\) (second-order pole) the residue
\[
\left. \left[\frac{(z^{2} - 1)^{2}}{z^{2} - 5z/2 + 1} \right] \right|_{z=0} = \frac{5}{2}
\]

by straightforward differentiation. Also,
\[
z^{2} - 5z/2 + 1 = (z - 2)(z - \frac{1}{2})
\]

and for the simple pole at \(z = \frac{1}{2}\) (inside the unit circle) we get the residue
\[
\left. \frac{(z^{2} - 1)^{2}}{z^{2}(z - 2)} \right|_{z=1/2} = \frac{9}{16} \cdot \frac{1}{(\frac{1}{2})(-\frac{1}{2})} = -\frac{3}{2}.
\]

14. Denote the integrand by \(f(z)\). The complex function \(f(z)\) has simple poles at \(z_{1} = e^{-\pi i/4}\) and \(z_{2} = e^{3\pi i/4}\) in the upper half-plane (and two further ones in the lower half-plane) with residues
\[
\frac{1 + i}{2\sqrt{2}(-1 + i)} = \frac{i}{\sqrt{8}} \quad \text{and} \quad \frac{1 - i}{2\sqrt{2}(1 + i)} = -\frac{i}{\sqrt{8}}
\]

respectively, as obtained from (4) in Sec. 16.3. Hence the answer is
\((-i/\sqrt{2})2\pi i = \pi \sqrt{2}.
\)