SOLUTIONS TO PROBLEM SET 14.3, page 663

2. The integrand is not analytic at ±1, z = 1 lies inside the contour, z = −1 outside. Hence, by Cauchy's formula, the integral has the value
\[ 2\pi i \cdot \frac{z^2}{z + 1} \bigg|_{z=1} = \frac{2\pi i}{2} = \pi i. \]

4. z = −1 lies inside C, z = 1 outside. Hence Cauchy's formula gives for
\[ \frac{z^2}{z^2 - 1} = \frac{z^2}{z - 1} \cdot \frac{1}{z + 1} \]
the value \[ 2\pi i \cdot \frac{z^2}{z - 1} \bigg|_{z=-1} = 2\pi i \cdot \left( -\frac{1}{2} \right) = -\pi i. \]
Note that by the deformation principle this must be the same value as in Prob. 1.

12. The integrand is
\[ \frac{z}{z^2 + 4z + 3} = \frac{z}{(z + 1)(z + 3)}. \]
z = −1 lies inside the contour, z = −3 outside. Accordingly, Cauchy's integral formula gives
\[ 2\pi i \cdot \frac{z}{z + 3} \bigg|_{z=-1} = 2\pi i \cdot \frac{-1}{2} = -\pi i. \]

SOLUTIONS TO PROBLEM SET 14.4, page 667

2. We obtain
\[ \frac{z^6}{(2z - 1)^6} = \frac{z^6}{2^6(z - \frac{1}{2})^6}. \]
Hence the solution is
\[ 2\pi i \cdot \frac{1}{2^6} \cdot \frac{z^6}{(z - \frac{1}{2})^6} \bigg|_{z=1/2} = 2\pi i \cdot \frac{1}{2^6} \cdot \frac{6!}{6} \bigg|_{z=1/2} = \frac{3\pi i}{32}. \]

4. We have to differentiate twice, obtaining
\[ \frac{2\pi i}{2!} (e^z \cos z)^n \bigg|_{z=\pi/4} = \pi i e^{\pi/4} (\cos z - \sin z)^n \]
\[ = \pi i e^{\pi/4} (\cos z - \sin z - \sin z + \cos z) \]
\[ = -2e^z \sin z. \]