SOLUTIONS TO PROBLEM SET 10.2, page 425

4. The exactness test for path independence gives

\( (F_2)_x = 8x e^{xy} = (F_1)_y \).

We find \( F = \text{grad} f \), where

\[ f(x, y) = x^2 e^{xy} \]

so that for the integral we have \( f(6, 1) - f(4, 0) = 36e^4 - 16e^0 = 1949 \).

6. For the \( dx \)-term and the \( dy \)-term, the exactness test of path independence gives \( 2xy e^{x^2 + y^2 + z^2} \), etc. We find

\[ f = \frac{1}{2} \exp (x^2 + y^2 + z^2) \).

Evaluation at the limits gives

\[ \frac{1}{2}(e^2 - 1) \).

SOLUTIONS TO PROBLEM SET 10.4, page 438

2. Integrate \( (F_2)_x - (F_1)_y = 2 - 12y \) over \( y \) from \(-2\) to \( 2 \) and the result over \( x \) from \(-2\) to \( 2 \), obtaining \((8 - 0) \cdot 4 = 32 \).

8. \( \mathbf{F} = \text{grad} (e^{-x} \cos y) \), so that the integral around a closed curve is zero. Also the integrand in (1) on the left is identically zero.

SOLUTIONS TO PROBLEM SET 10.5, page 442

2. Circles, straight lines through the origin. A normal vector is

\[
N = \begin{vmatrix}
i & j & k \\
\cos v & \sin v & 0 \\
-u \sin v & u \cos v & 0 \\
\end{vmatrix} = [0, 0, u] = uk.
\]

At the origin this normal vector is the zero vector, so that (4) is violated at \((0, 0)\). This can also be seen from the fact that all the lines \( v = \text{const} \) pass through the origin, and the curves \( u = \text{const} \) (the circles) shrink to a point at the origin. This is a consequence of the choice of the representation, not of the geometric shape of the present surface (in contrast with the cone, where the apex has a similar property, but for geometric reasons).

4. The parameter curves \( u = \text{const} \) are ellipses, namely, the intersections of the cylinder with planes \( z = \text{const}; \) \( a \) and \( b \) are their semi-axes. The curves \( v = \text{const} \) are the generating straight lines of the cylinder, which are perpendicular to the \( xy \)-plane. A normal vector is \([-b \cos v, \ -a \sin v, \ 0]\). Note that these normal vectors are parallel to the \( xy \)-plane, which is geometrically obvious.

For \( b = a \) we obtain the representation in Example 1 of the text.

Note further that the normal vectors are independent of \( u \); they are parallel along each generator \( v = \text{const} \), which is also geometrically obvious.

6. \( z = \arctan (y/x) \), helices (hence the name!), horizontal straight lines. This surface is similar to a spiral staircase, without steps (as in the Guggenheim Museum in New York). A normal vector is

\[
[sin v, \ -cos v, \ u].
\]