ESE 524. Detection and Estimation Theory

R. Martin Arthur

Tu/Th 2:30PM-4:00PM in Cupples II 220

***Abstract*— ESE 524: Study of detection, estimation and modulation theory, detection of signals in noise, estimation of signal parameters, linear estimation theory. Kalman-Bucy and Wiener filters, nonlinear modulation theory, optimum angle modulation. Prerequisite: ESE 520.**

ESE 520 ⇒ Review of probability theory, models for random signals and noise, calculus of random processes, noise in linear and nonlinear systems, representation of random signals by sampling and orthonormal expansions. Poisson, Gaussian, and Markov processes as models for engineering problems. Prerequisite: ESE 326.

ESE 326 ⇒ Study of probability and statistics together with engineering applications. Probability and statistics: random variables, distribution functions, density functions, expectations, means, variances, combinatorial probability, geometric probability, normal random variables, joint distribution, independence, correlation, conditional probability, Bayes theorem, the law of large numbers, the central limit theorem. Applications: reliability, quality control, acceptance sampling, linear regression, design and analysis of experiments, estimation, hypothesis testing. Examples are taken from engineering applications. Prerequisites: Math 233 or equivalent.

I. Lecture #1

DETECTION and estimation ...

R. M. Arthur

TAKE THE ROLL !!!

1) Website
2) Syllabus [Text(s)], Grading & HW#1
3) Overview Slides
4) Applications
   - Central limit demonstration
     → Run centrallimit.m in /mfls
   - Inverse ECG problem
     → a7ese524-ecg.ppt in /lecture/pptslds
   - Tikhonov regularization
     → memcreso.pdf in /appls
   - Magnitude of the multipole moment
     → quadmom.pdf in /appls
   - Probabilistic image model & Inference of pose
     → c:/1a/text/web/pbuim

⇒ NO PREREQUISITE QUIZ

R. M. Arthur is with the Department of Electrical and Systems Engineering, Washington University School of Engineering. e-mail: rma@ese.wustl.edu

II. Lecture #2

Comments/Questions
- course content
- organization
- homework
- other

Chapter 1 Exposition

→ Touchtone.pdf detector

1) Detection example
   → Run corrconv.m in /mfls
2) Estimation example

III. Lecture #3

⇒ Homework format, [example]

Homework problem 1.1

Problem Statement: Given that the symbol of the detector in section 1.3 is known and has energy E_s and that the noises are iid Gaussian RV(variables)s

a) Find the distribution of the correlation statistic under H_0 and H_1

b) If the SNR is the mean squared divided by the variance, show that $SNR = (\mu^2/\sigma^2)E_s$

→ Run P1_1.m in /mfls

1) Estimation example
   → Run recurest.m in /mfls
2) Notation & Terminology
   - Probability Distributions
   - Linear Models → ARMA example 1.3

IV. Lecture #4

Go over Homework #2

LINEAR ALGEBRA

1) Vector Spaces
   - Euclidean and Hilbert Spaces
   - Matrices
     → Run dieroll in /mfls
     → LSE from Art68, p13
     → Run mvn in /mfls
2) Linear Independence
   - Gram Determinant
   - Sequences of Gram Determinants
   - SVD (Help on rank [doc])
     → sw_art2009icbem-rmas.ppt

V. Lecture #5

Homework #1 grades (hi=100, low=80, μ=93, σ=7)
Answers online with password

1) Linear Independence
   - Cholesky & QR Factors
   - Gram-Schmidt Procedure to find $\mathbf{Q} = \mathbf{X}\mathbf{R}^{-1}$
   - Matrix rank via SVD
     → /mfls/qrfactors.m
2) Linear Subspaces
   - Basis for Subspaces
   - Uniqueness
   - Dimension, Rank, and Nullity
   - Linear Equations & Decomposition of R^n
3) Hermitian Matrices
   - Eigenvalues are real and orthogonal
   - Matrices are diagonalizable
4) Singular Value Decomposition

VI. Lecture #6

Password
Go over Homework #3

1) Normal Distribution
   Nova: Wisdom of the Crowds
   → http://www.youtube.com/watch?v=Z82B1zsvyZU
   - Multivariate → /appls/momtof-charfun.pdf
   - Bi-Variate
   - Transformations, Example: χ^2 to Rayleigh
     → /mfls/chi2rayl.m
2) Sufficiency
   - Bernoulli trials
     → /mfls/berntrl.m
   - Factorization Theorem

VII. Lecture #7

Homework #2 grades (hi=100, low=90, μ=98, σ=3)

1) Sufficiency: Definition
   → /mfls/berntrl.m
2) Factorization Theorem
   - Discrete variables
   - Continuous RVs
     → /mfls/chi2raly.m
3) Complete Sufficient Statistics
   - Unbiased
   - Unique
   - Minimal

VIII. Lecture #8

Go over Homework #4 and QUIZ
Go over reference list in handouts

1) Complete Sufficient Statistics
   → suffinvar.pdf
   - Unbiased
   - Unique
   - Minimal
2) Minimum Variance Unbiased Estimators
3) Exponential Families
   - Definition
   - Sufficient Statistic
   - k-parameter family
   - two-parameter normal pdf example

IX. Lecture #9

Homework #3 grades
→ vantrees-suffstat.pdf

1) Hypothesis Testing
   - Normal example as likelihood rationale
   - Classification: Simple or multiple vs Composite
   - Binary:
     Type I error or false alarm with size α or P_{FA}
     Type II error or miss $1-\beta$. β is the power or P_D
2) Receiver Operating Characteristics
   - Plots of β vs α, "chance line"
   - ARGUS example
3) Baysean vs Neyman-Pearson Approaches
4) Digital communications example
5) Neyman-Pearson Lemma

X. Lecture #10

Homework #3 grades (hi=98, low=86, μ=95, σ=4)
Run → /mfls/npnormal.m
Run → /saecg/fmagctle.m with sw=4

1) Application of Neyman-Pearson Lemma
2) Likelihood ratio and ROCs
   - Poisson example
   - Gaussian examples

XI. Lecture #11

1) HW#4, Problem 1 Setup
   → /mfls/hw4p1.m
2) Scalar and Vector Gaussian examples with different means
   - Scalar
   - Vector
     - Dependent samples
     - Independent samples
       $\mathbf{R} = \mathbf{I}$, orthogonal and antipodal examples

   Run (in /mfls/) → rocgd.m & erfcdemo.m

XII. Lecture #12

Go over Homework #5
Homework #4 problems
→ /mfls/hw4p1b.m
→ /mfls/hw4p3.m
Show hw4p3 ROCs for SNR dependence
Composite Hypotheses

- Uniformly Most Powerful Test
- Karlin and Rubin Theorem

(Connect scalar & vector Gaussian examples

- Scalar Gaussian detection
- Vector Gaussian detection

⇒ Administer Quiz #1

XIII. Lecture #13

Homework #4 grades (hi=100, low=70, μ=94, σ=9)
Go over Quiz #1
→ /mfls/quizsb0.m
→ /mfls/hw4p2.m
Ben Slocumb (pdf) from Numerica visit
Karlin and Rubin Theorem
Complete Gaussian examples:
$\mathbf{R} = \mathbf{I}$ orthogonal and antipodal examples
Sufficiency & Invariance

1) Likelihood ratio
2) UMP
3) Invariance

XIV. Lecture #14

Bayesian Detection

1) Invariance
2) Example 4.6 / 4.9
   → /mfls/scharfex49.m
3) Reduction → suffinvar.pdf in handouts
4) Bayes Principle
5) Machine vision example
6) Loss and Risk
7) Two person game
8) Weighted likelihoods

XV. Extra Session - Test Review

Monday 3/15, Bryan 305, 5:30-6:30PM

Normal Matched Filter
Gaussian equal means, different covariance example
→ /mfls/roggddc.m

XVI. Lecture #15

Homework #5 grades (hi=100, low=57, μ=90, σ=11)
Problem 4.5c
Temperature Estimation example

Bayesian Detection

1) Bayes Risk
2) Simple binary hypotheses

XVII. Session #16 - TEST (3/18)

XVIII. Lecture #17

Bayesian Detection

Set up Homework problem 5.14 in Homework #6

1) Simple binary hypotheses
   - Gaussian example
2) Posterior distributions
3) Detection and sufficiency

XIX. Lecture #18

Go over Homework #7 (ese524hw7_p6-101720.pdf)

- Binary decisions: N-P & Bayes

Bayesian Detection

- Multiple hypotheses
- Maximum likelihood detection
- Minimax Bayes decisions

→ /mfls/bayesdemorma.m & → /mfls/bayesenv.m

XX. Lecture #19

Test scores → High 95, Low 74, Mean 85, STD 6

Maximum Likelihood Estimation

1) Maximum Likelihood Principle
2) ML Estimates
   - Coin toss example
   - Empirical distributions
   - ML estimator for μ and σ from $X : N[\mu, \sigma^2]$
   - Sufficiency

XXI. Lecture #20

"Best" Estimators
→ /mfls/normmnsg.m

- Score function
- Cramer-Rao Lower Bound
- Fisher Information Matrix
- Efficiency
- Asymptotic properties

XXII. Lecture #21

Go over Homework #8

Maximum Likelihood Estimators

1) Multivariate normal distributions
   - ML Estimator
   - CR bound and FIM computation
2) Minimum-variance unbiased estimators
3) Linear statistical models
   - ARMA parameters
   - Signal subspace

XXIII. Lecture #22

Homework Scores
Linear statistical models
→ tikcreso.pdf

Bayesian Estimators

1) Bayes risk
   - Square-error loss
   - Absolute-error loss
   - Uniform-cost loss
2) MAP estimates

XXIV. Lecture #23

Homework #7 grades (hi=100, low=80, μ=92, σ=5)

Bayesian Estimators

1) Posterior estimates
2) Conjugate priors
3) Recursive estimation
   - Markov random process
   - Update / Propagate

XXV. Lecture #24

Kalman Filters

1) State space
2) The Bayes approach

Wiener Filters

- Convolution
- Deconvolution
  → run diagwft.m in mfls

XXVI. Lecture #25

Go over Homework #9
Spectral Noise and Power from Schraf p. 312

Wiener Filters

- Least-squares residual
- Linear constraint operators
  → run wf.m in mfls
  → run rest524.m in mfls

Kalman Filters

Simple Kalman example: motion in two coordinates

$$\mathbf{x}_{t+1} = A_t \, \mathbf{x}_t + \mathbf{w}_t$$
$$\mathbf{y}_t = C_t \, \mathbf{x}_t + \nu_t$$

→ run kalmanexample.m in mfls
→ show kalmanbayes.m on website

XXVII. Lecture #26

Minimum Mean-Squared Error Estimators

1) Conditional Expectation and Orthogonality
2) LMMSE Estimators
3) Low Rank Wiener Filters

XXVIII. Lecture #27

Least Squares Estimators

1) Linear Models
2) Least-Squares Solutions and Projections
3) Inference of heart-surface potentials

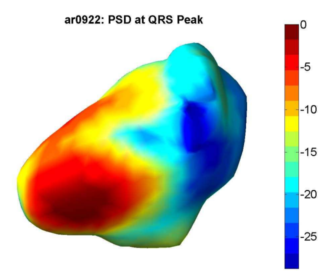

Fig. 1. Inferred power spectral density map on a normal subject

Power point slides on inverse electrocardiology
→ run runsvd1.m
→ run bfp3dl3k.m, load ar1018bip, and run bflyppot.m

XXIX. Lecture #28

Homework #8 grades (hi=98, low=83, μ=95, σ=5)
Course evaluation ⇒ http://evals.wustl.edu/
EM algorithm†
†Dempster et al., J Royal Stat Soc, 39:1-38, 1977.

XXX. Exam Review: Sunday 5/9, Bryan 305, 2-3:00PM