ESE 520. Probability and Stochastic Processes

R. Martin Arthur

Tu/Th 4:00PM to 5:30PM in Eads 103

Abstract—ESE 326 ⊆ Study of probability and statistics together with engineering applications. Probability and statistics: random variables, distribution functions, density functions, expectations, means, variances, combinatorial probability, geometric probability, normal random variables, joint distribution, independence, correlation, conditional probability, Bayes theorem, the law of large numbers, the central limit theorem. Applications: reliability, quality control, acceptance sampling, linear regression, design and analysis of experiments, estimation, hypothesis testing. Examples are taken from engineering applications. Prerequisites: Math 233 or equivalent.

ESE 520: Review of probability theory, models for random signals and noise, calculus of random processes, noise in linear and nonlinear systems, representation of random signals by sampling and orthonormal expansions. Poisson, Gaussian, and Markov processes as models for engineering problems. Prerequisite: ESE 326.


I. Lecture #1

Probability and stochastic processes ...

R. M. Arthur

TAKE THE ROLL !!!

1) Website
2) Syllabus
3) Text(s)
4) Grading & Homework (QUIZ) #1
5) Overview Slides

⇒ PREREQUISITE QUIZ
⇒ Answers: On-line Video

6) Applications
   • Central limit demonstration
     ⇒ Run centrallimit.m in /mfls
   • pdfs of RF and B-Mode ultrasonic images
     ⇒ usipmfs-b0809.pptx in /lecture/pptsls
   • Cardiac source estimation
     ⇒ b0520-ecg.pptx in /lecture/pptsls
   • Moments of a TOF surface
     ⇒ tofmom-b0809.pptx in /lecture/pptsls
   • Probabilistic image model & Inference of pose
     ⇒ c:/1a/text/web/pbuim

II. Lecture #2

⇒ Comments / Questions
• course content
• organization
• homework
• other

Probability Models
1) Random experiment
2) Statistical Regularity
3) Relative Frequency
4) Axiomatic Approach
5) Building a Model

Basic Concepts
1) Random Experiments

III. Lecture #3

⇒ Comments or questions

Basic Concepts
1) Random Experiments
2) Axioms
   ⇒ Ex 2.10
   ⇒ Homework/Quiz format, [Ex 2.13]
3) Probability from Counting
4) Conditional Probability
   ⇒ Homework/Quiz format, [Ex 2.28]
   ⇒ Run ex228.m in /mfls

IV. Lecture #4

⇒ Comments or questions
Prerequisite Quiz grades (hi=93, μ=34, σ=22)
Go over Homework #2

Basic Concepts
1) Independence
2) Sequential Experiments
   • Multinomial probability law
   • Markov chains

⇒ Administer Quiz 1
V. Lecture #5
Go over Quiz #1 Solution and scores.
Hi 108, Low 84 , $\mu =100$, $\sigma = 8$. Password
⇒ Comments or questions

Discrete Random Variables
1) Random Variables
2) Discrete RVs and pmfs
3) Expected Value & Moments of DRVs
4) Conditional Probability Mass Functions
5) The Poisson RV

VI. Lecture #6
Go over Homework #3
⇒ Comments or questions

Concluding remarks on discrete RVs
• → Run poifig.m in /mfls
• ex330.pdf in /lecture

Probability Models
1) Cumulative Distribution Function
2) Probability Density Function

⇒ Administer Quiz 2

VII. Lecture #7
Go over Quiz #2 (hi=100, low=80, $\mu=89$, $\sigma=6$)
⇒ Comments or questions

1) Probability Density Function
   • Ex. 4.1
   • Ex. 4.9
   • Ex. 4.11
2) Expected Value
3) Important Continuous RVs
   • Ex. 4.32

VIII. Lecture #8
Go over Homework #4
⇒ Comments or questions

1) Functions of a RV
2) Markov and Chebyshev Inequalities
3) Transform Methods
   • Ex. 4.43
4) Entropy

⇒ Administer Quiz 3

IX. Lecture #9
Go over Quiz #3 (hi=100, low=76, $\mu=92$, $\sigma=7$)
⇒ Comments or questions

1) Transform Methods
   • Ex. 4.43
   • Ex. 4.44
2) Entropy

Vector Random Variables
1) Properties of Vector RVs
   • Ex. 6.2

X. Lecture #10
⇒ Comments or questions

1) Properties of Vector RVs
   • Ex. 6.6
   • Run jgrv.m in /mfls
   • Ex. 5.18
2) Functions of RVs
   • Ex. 6.15

XI. Lecture #11
Go over Homework #4
⇒ Comments or questions

1) Expected Values of Vector RVs
   • Ex. 6.16
   • Ex. 6.19

⇒ Administer Quiz 4

XII. Lecture #12
The textbook is on Reserve at Olin
⇒ Comments or questions

1) Expected Values of Vector RVs
   Run diagcov.m in /mfls
2) Jointly Gaussian RVs
   • Ex. 6.20

Sums of Random Variables & Long-Term Averages
1) Sum of RVs
   • Ex. 7.1
   • Ex. 5.39
   • Ex. 7.3
XIII. Lecture #13
Go over Quiz #4 (hi=100, low=72, \( \mu = 92 \), \( \sigma = 8 \))
⇒ Comments or questions

1) Sample Mean
   • Weak Law
   • Strong Law
2) Central Limit Theorem
   • Ex. 7.3
   • Ex. 7.11
   Run → centrallimit.m in /mfls
   Run → ex711.m & erfcdemo in /mfls

XIV. Lecture #14
⇒ Comments or questions
Help session & Test
1) Convergence of RV Sequences
   • Ex. 7.18

⇒ Administer Quiz 5

XV. Help Session
Sunday 10/17, Bryan 305, 2-3PM
Test 10/21, Bryan 305, 4-5:30 PM

XVI. Lecture #15
⇒ Comments or questions
Central Limit Theorem
Run → centrallimit2.m in /mfls

Random Processes
1) Definition
   Ex 9.1
   Ex. 9.3
2) Specification
   Ex 9.8

XVII. Session #16 - TEST (10/21)

XVIII. Lecture #17
Go over Quiz #5 (hi=100, low=72, \( \mu = 88 \), \( \sigma = 8 \))
Go over the Test (hi=100, low=45, \( \mu = 72 \), \( \sigma = 16 \))
Run → ese520fb0scores.m in /test
Run → ese520fb0test.m in /mfls
Go over Homework #6

⇒ Comments or questions
http://academicearth.org/lectures/digital-communication-random-processes
1) Specification
   Ex 9.8
   Ex. 9.12
2) Discrete-Time Processes
   Ex 9.13

XIX. Lecture #18
⇒ Comments or questions
Ex. 9.12
1) Discrete-Time Processes
   Ex. 9.13
   http://research.engineering.wustl.edu/~dls/bio.html
   Random Point Processes in Time and Space (Springer Texts in Electrical Engineering) by Donald L. Snyder and Michael I. Miller, 1991. On Amazon 1 used from $546.31

XX. Lecture #19
⇒ Comments or questions
1) Discrete-Time Processes
   Run → walkproc.m in /mfls
2) Poisson Processes

⇒ Administer Quiz 6

XXI. Lecture #20
⇒ Comments or questions
1) Poisson Processes
   Ex. 9.21
2) Gaussian Processes
   Ex. 9.27
   Ex. 9.28
3) Wiener Process
XXII. Lecture #21

⇒ Comments or questions

1) Stationary Random Processes
   Ex. 9.38
2) Continuity, Derivatives & Integrals of RPs
   • Continuity
     Ex. 9.40

XXIII. Lecture #22

Go over Quiz #6 (hi=100, low=72, μ=92, σ=8)
Go over Homework #8

1) Continuity, Derivatives & Integrals of RPs
   • Derivatives
     • Integrals
2) Time Averages and Ergodic Theorems

⇒ Administer Quiz 7

XXIV. Lecture #23

Analysis and Processing of Random Signals

1) Power Spectral Density
   • Continuous- & Discrete-Time RPs
     Fig 10.3 PSD of MA
     Ex. 1007
   • PSD as a Time Average
2) Response of Linear Systems to Random Signals

⇒ Administer POP Final

XXV. Lecture #24

1) Response of Linear Systems to Random Signals
   • Continuous-Time
     Ex. 10.14
Solve POP FINAL

XXVI. Lecture #25

Go over Quiz #7 High=100, Low=72, μ=88 σ=4
Go over Homework #9
Solve POP FINAL
1) Bandlimited Random Processes
   Fig. 10.10
   Run → demontr.m & demontrs.m in /mfls
2) Optimum Systems

⇒ Administer Quiz #8

XXVII. Lecture #26

Markov Chains

1) Markov Processes
   Ex. 11.02
2) Discrete-Time Markov Processes
   Ex. 11.07
   • n-Step Transition Probabilities
   • State Probabilities
     Ex. 11.10
   • Steady-State Probabilities
     Ex. 11.11
     Run → ex1102.m & ex1110.m in /mfls

XXVIII. Lecture #27

Go over Quiz #8 High=100, Low=64, μ=84 σ=8

1) Discrete-Time Markov Processes
   Ex. 11.10
   • Steady-State Probabilities
     Ex. 11.11
     Run → ex1102.m & ex1110.m in /mfls
2) Classes of States
   • Recurrence Properties
   • Limiting Probabilities

XXIX. Lecture #28

Course evaluation ⇒ http://evals.wustl.edu/

1) Classes of States
   • Recurrence Properties
   • Limiting Probabilities
     Theorems 1 & 2
     Ex. 11.12
     Ex. 11.30
     Run → ex1112.m in /mfls

⇒ Administer Quiz 9

XXX. Lecture #29

Course evaluation ⇒ http://evals.wustl.edu/

Exam Review: Sunday 12/12, Bryan 305, 2-3:00PM
Final Exam: Thursday 12/16, Bryan 305, 4-6:00PM

1) Continuous-Time Markov Chains
   • State Occupancy Times
   • Transition Rates
   • Steady-State and Global Balance
   • Limiting Probabilities

Solve POP FINAL part b