Merrill Lynch Improves Liquidity Risk Management for Revolving Credit Lines

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Merrill Lynch Bank USA has a multibillion dollar portfolio of revolving credit-line commitments with over 100 institutions. These credit lines give corporations access to a specified amount of cash for short-term funding needs. A key risk associated with credit lines is liquidity risk, or the risk that the bank will need to provide significant assets to the borrowers on short notice. We developed a Monte Carlo simulation to analyze liquidity risk of a revolving credit portfolio. The model incorporates a mix of OR/MS techniques, including a Markov transition process, expert-system rules, and correlated random variables to capture the impact of industry correlations among the borrowers. Results from the model enabled the bank to free up about $4 billion of liquidity. Over the 21 months since the bank implemented the model, the portfolio has expanded by 60 percent to over $13 billion. The model has become part of the bank’s tool kit for managing liquidity risk and continues to be used every month.

Key words: financial institutions: banks; probability: Markov processes.

Merrill Lynch was founded in 1914 by Charles E. Merrill, with the vision of bringing Wall Street to Main Street. Merrill Lynch has two major business divisions, the Global Private Client Group (GPC) and Global Markets and Investment Banking (GMI).

GPC is the retail part of the business, providing brokerage, investment, and banking services to individual retail clients and small to midsize businesses. Merrill Lynch provides investment services to these clients through a sales force of 14,000 financial advisors (FAs) in 500 offices around the world. GPC supports trading in a wide range of securities, including stocks, bonds, mutual funds, and other financial instruments. It also offers such products as mortgages, home equity loans, annuities, and insurance and such
services as trust and estate-planning services, and 401(k) employee-benefits administration. GPC serves about 3 million households and has $1.3 trillion of assets under management.

GMI serves major corporations and institutions around the world. It helps companies raise capital through new issues of equity and debt, and it acts as a market maker in a wide range of securities. GMI also provides advice and guidance to companies in such areas as mergers and acquisitions.

Merrill Lynch established its management science group in 1986, and the group has been part of the GPC organization since 1990. It helped Merrill Lynch win the INFORMS Prize in 1997 for the effective integration, application, and impact of management science on the decision making and success of the firm. In 2001, the group won the Edelman Prize in recognition of the pricing analysis for Integrated Choice, a new product at Merrill Lynch, which gathered $83 billion of client assets during its first 18 months.

The mission of the management science group is to use quantitative modeling and analysis to support strategic decision making in complex business situations. The group provides analytical and business consulting to many GPC business units using a broad range of operations research and management science techniques, including optimization, simulation, and multivariate statistics. Application areas include compensation analysis, pricing analysis, mutual-fund portfolio optimization, quantifying the impact of business strategies, evaluating the effectiveness of marketing efforts, and developing prospecting and cross-selling models.

The Merrill Lynch banking group supports both GPC and GMI. It comprises several Merrill Lynch affiliates, including Merrill Lynch Bank USA (ML Bank USA) and Merrill Lynch Bank and Trust Co. (MLB&T). ML Bank USA has assets of over $60 billion. The bank acts as an intermediary, accepting deposits from Merrill Lynch retail customers and using the deposits to fund loans and make investments. Uninvested cash in Merrill Lynch investment accounts can be automatically swept into ML Bank USA deposits. One of the ways ML Bank USA deploys these assets is by providing revolving credit lines to institutional borrowers. Currently ML Bank USA has a portfolio of about $13 billion in credit-line commitments with over 100 institutions.

In the context of this work, liquidity is the ability to meet all cash obligations when due. For a bank, such obligations are caused by the demand for funds made by its borrowers or by its depositors. This concept of liquidity is distinct and unrelated to liquidity in the trading markets (Schomburg 2001). Consequently, liquidity risk is a bank’s potential inability to meet its cash obligations. The Merrill Lynch Banking Group seeks to ensure liquidity at all times, across market cycles, and through periods of financial stress. It does so by maintaining positive cash flow gaps over multiple years in a simulated contingency scenario. It periodically tests the availability, under stress conditions, of alternative sources of funding, including the Federal Reserve, Federal Home Loan Banks, and repurchase agreements (a repurchase agreement is a contract in which the seller of securities agrees to buy them back at a specified time and price).

**Background: Revolving Credit Lines**

Revolving credit lines, or credit facilities, give borrowers access to a specified amount of cash on demand for short-term funding needs. These credit lines function for corporations in the same way credit cards or home equity lines of credit work for individuals. The bank establishes credit limits and allows the corporations to borrow, on demand, up to these limits (Figure 1). The borrowers are institutional clients who need short-term funding. On the other side are the individual clients with investment accounts at Merrill Lynch. ML Bank USA acts as an intermediary, using the cash deposits to fund the revolving credit lines.

The credit lines banks offer are not a primary source of short-term borrowing but serve mainly as backup liquidity for issuers of commercial paper, which is the cheapest source of short-term borrowing for corporations and foreign governments. They typically use these lines to retire maturing commercial paper during the process of rolling it over, or when they cannot roll it over because of a credit downgrade or general adverse market conditions. Rating agencies require evidence of short-term backup liquidity before assigning a commercial paper rating to an issuer. They typically require liquidity backup of about 50 percent of commercial paper outstanding for the highest-rated
issuers and typically 100 percent for those with less than the highest A1-P1 rating. (Ratings for short-term debt are issued by all four US credit-rating agencies. Standard and Poor's short-term ratings range from A-1 (highest) to D (default); Moody's range from P-1 (highest) to P-3 (lowest); they also issue subgrades, for example, A-1+) (Hahn 1993). So, in addition to serving as liquidity backup, revolvers play a credit-enhancing role (Federal Reserve System Board of Governors 2003).

Each credit line has an associated dollar amount, an expiration date, and renewal options. Some borrowers have multiple lines, or tranches, with different dollar amounts and expirations. When companies have multiple lines, they can choose to use any one tranche or a combination of tranches. Once a credit line is in use, the borrower can choose to continue to use the line up to its limit or pay it off. The borrower must immediately repay any amounts outstanding when the credit line expires (Table 1).

Regulatory treatment of revolvers depends on their original term to maturity. Regulatory capital is needed (at a 50 percent risk weight) only if the revolving credit facility has a maturity of a year or more. For this reason, banks typically offer facilities consisting of two tranches: a low-priced 364-day tranche and a higher-priced one maturing in three to five years or more.

Credit lines are often funded through a consortium (syndicate) of banks, which helps spread the liquidity

<table>
<thead>
<tr>
<th>Facility borrower</th>
<th>Facility name</th>
<th>Facility effective date</th>
<th>Facility maturity date</th>
<th>Commitment amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversified/Conglomerate</td>
<td>Revolver 5 year</td>
<td>7/8/2002</td>
<td>7/8/2007</td>
<td>60,000,000</td>
</tr>
<tr>
<td>services 1</td>
<td>Revolver 364D-1 year term out</td>
<td>3/31/2003</td>
<td>1/31/2006</td>
<td>55,000,000</td>
</tr>
<tr>
<td>Insurance 1</td>
<td>Revolver 3 year-2 year term out</td>
<td>7/17/2003</td>
<td>7/15/2004</td>
<td>50,000,000</td>
</tr>
<tr>
<td>Insurance 2</td>
<td>Revolver 364D-1 year term out</td>
<td>7/18/2002</td>
<td>7/18/2007</td>
<td>65,000,000</td>
</tr>
<tr>
<td>Utilities 1</td>
<td>Revolver 364D-1 year term out</td>
<td>3/7/2003</td>
<td>4/7/2004</td>
<td>350,000,000</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 4 year</td>
<td>3/7/2002</td>
<td>6/22/2006</td>
<td>50,000,000</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 364D-1 year to 2003</td>
<td>6/20/2003</td>
<td>6/18/2004</td>
<td>31,818,182</td>
</tr>
</tbody>
</table>

Table 1: These sample credit facilities range in duration from one to five years and represent commitment amounts of $10 to 350 million. A number of borrowers have multiple facilities (Diversified/Conglomerate 1, Insurance 2, and Retail 1).
risk across multiple institutions (Table 2). The bank generally determines the lending rate on these loans as a floating base rate tied to the prime rate or to the London Interbank Offered Rate (LIBOR). It typically negotiates the spread over the base rate when it establishes the credit line (Hahn 1993).

Some revolving credit lines have a term-out option, which allows the borrower to use the funds for an additional period after expiration, typically for one year. The bank negotiates this feature with the borrower when it establishes the credit line, typically for lines that expire in less than one year. If the borrower chooses to invoke a term-out option, the bank cannot refuse the request. At the end of the additional period, the company must fully repay any outstanding amount (Table 3).

The bank can renew credit lines, typically basing its decisions on the credit rating of the borrower when the line expires, the history of the borrower’s use of the credit line, and other revolving credit-line commitments at that time (Table 4).

Banks’ ability to offer revolving credit facilities and other forms of backup liquidity to commercial-paper issuers becomes most crucial in times of systemwide liquidity shocks (Gatev and Strahan 2005, Kanatas 1987, Kashyap et al. 2002). Gatev and Strahan contend that banks’ deposit inflows provide a natural

<table>
<thead>
<tr>
<th>Facility name</th>
<th>Commitment amount</th>
<th>Global facility commitment amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolver 5 year</td>
<td>60,000,000</td>
<td>6,000,000,000</td>
</tr>
<tr>
<td>Revolver 364D-1 year term out 2003</td>
<td>25,000,000</td>
<td>2,000,000,000</td>
</tr>
<tr>
<td>Revolver 3 year-2 year term out</td>
<td>55,000,000</td>
<td>500,000,000</td>
</tr>
<tr>
<td>Revolver 364D-1 year term out 2003</td>
<td>50,000,000</td>
<td>1,375,000,000</td>
</tr>
<tr>
<td>Revolver 5 year</td>
<td>65,000,000</td>
<td>1,375,000,000</td>
</tr>
<tr>
<td>Revolver 364D-1 year term out</td>
<td>10,000,000</td>
<td>225,000,000</td>
</tr>
<tr>
<td>Term 364D-1 year term out</td>
<td>350,000,000</td>
<td>350,000,000</td>
</tr>
<tr>
<td>Revolver 4 year</td>
<td>50,000,000</td>
<td>2,250,000,000</td>
</tr>
<tr>
<td>Revolver 364D-1 year to 2003</td>
<td>31,818,182</td>
<td>1,750,000,000</td>
</tr>
</tbody>
</table>

Table 2: These examples show credit amounts committed by one bank versus the total commitment by the syndicate of banks for each facility in our sample set.

<table>
<thead>
<tr>
<th>Facility borrower</th>
<th>Facility name</th>
<th>Facility effective date</th>
<th>Facility maturity date</th>
<th>Final term-out date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversified/Conglomerate services 1</td>
<td>Revolver 5 year</td>
<td>7/8/2002</td>
<td>7/8/2007</td>
<td>(Null)</td>
</tr>
<tr>
<td>Diversified/Conglomerate services 1</td>
<td>Revolver 364 days-1 year term out 2003</td>
<td>7/7/2003</td>
<td>7/6/2004</td>
<td>7/6/2005</td>
</tr>
<tr>
<td>Insurance 2</td>
<td>Revolver 5 year</td>
<td>7/18/2002</td>
<td>7/18/2007</td>
<td>(Null)</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 4 year</td>
<td>3/7/2002</td>
<td>6/22/2006</td>
<td>(Null)</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 364 days-1 year to 2003</td>
<td>6/20/2003</td>
<td>6/18/2004</td>
<td>6/18/2005</td>
</tr>
</tbody>
</table>

Table 3: In these examples of term-out conditions in our sample facilities, three of the facilities have no term-out option, and all but one (Insurance 1) are associated with facility durations of less than one year.

<table>
<thead>
<tr>
<th>Facility borrower</th>
<th>Facility name</th>
<th>Ratings current S&amp;P</th>
<th>Ratings current Moody's</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversified/Conglomerate services 1</td>
<td>Revolver 5 year</td>
<td>BBB+</td>
<td>Baa1</td>
</tr>
<tr>
<td>Diversified/Conglomerate services 1</td>
<td>Revolver 364 days-1 year term out 2003</td>
<td>BBB+</td>
<td>Baa1</td>
</tr>
<tr>
<td>Insurance 1</td>
<td>Revolver 3 year-2 year term out</td>
<td>AAA</td>
<td>Aaa</td>
</tr>
<tr>
<td>Insurance 2</td>
<td>Revolver 364 days-1 year term out 2003</td>
<td>AAA</td>
<td>Aaa</td>
</tr>
<tr>
<td>Insurance 2</td>
<td>Revolver 5 year</td>
<td>AAA</td>
<td>Aaa</td>
</tr>
<tr>
<td>Utilities 1</td>
<td>Revolver 364 days-1 year term out</td>
<td>BBB−</td>
<td>Baa2</td>
</tr>
<tr>
<td>Utilities 2</td>
<td>Term 364 days-1 year term out</td>
<td>BB−</td>
<td>Ba1</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 4 year</td>
<td>AA</td>
<td>Aa2</td>
</tr>
<tr>
<td>Retail 1</td>
<td>Revolver 364 days-1 year to 2003</td>
<td>AA</td>
<td>Aa2</td>
</tr>
</tbody>
</table>

Table 4: This table shows the current credit ratings for sample facilities provided by two rating agencies with their different scales.
hedge against increases in loan demand that follow declines in market liquidity, for example, during the recent Enron crisis (Zuckerman 2002), and thus banks can provide backup liquidity at a lower cost than other financial institutions. Empirical evidence supports the argument that when market liquidity dries up and commercial paper rates rise, the increased deposit inflows associated with the flight to cash safety that banks experience helps them to meet increased demand for loans from borrowers drawing down their preexisting commercial paper backup lines without having to tap into their holdings of liquid assets.

Two key drivers of credit-line usage could affect liquidity risk: (1) low-level ongoing use by the various borrowers for short periods and (2) default by companies to whom a bank has extended credit, a greater threat. The bank considers a borrower to be in default when the borrower fails to make an interest or principal payment on any debt obligation on time. That failure signals severe financial distress and causes an immediate downgrade to the lowest credit rating.

As a borrower’s credit rating worsens, it tends to increase the use of its revolving credit facilities. Because a credit rating of A or better is required to gain access to the commercial paper market, borrowers rated below A (Table 5) use revolving credit as a substitute to commercial paper for short-term funding. A downgrade to a rating below investment grade typically triggers debt covenants that call for the accelerated repayment of certain types of a borrower’s debt obligations. In this case, a borrower increases revolver use to obtain the funds to meet such obligations. Usage jumps in the event of default, as borrowers tap into every source of liquidity available (Marker 1997). A summary of revolver usage statistics compiled by Asarnow and Marker (1995) based on monthly Citibank portfolio data from 1987 through 1993 (Table 6) gives evidence of these trends. Furthermore, a number of companies or industries may experience financial weakness simultaneously and invoke the use of their credit lines, creating a drain on liquid funds. In these situations, the companies are unlikely ever to repay the loans. The bank must manage deposits to meet these liquidity risks as well as the overall likelihood of defaults in the revolving credit portfolios.

Revolving credit facilities generate revenues in the form of interest and fees. Borrowers pay facility fees regardless of usage. The bank assesses a commitment fee ranging from five to more than 50 basis points annually on the unused portion of a facility. It charges borrowers that use their facilities heavily a usage fee. The cash committed to funding the facility amounts to a fraction of the facility size except during occurrences of extreme company-specific or marketwide liquidity stress.

**Business Objectives**

In 2000, ML Bank USA began establishing revolving credit lines for client companies to use excess liquidity profitably. Although the business began slowly, the bank expected it to grow rapidly over the next several years.
years. Initially the bank developed its own spreadsheet model to assess the liquidity risk, but it soon recognized that it needed a more comprehensive and sophisticated model to help it to manage the growing portfolio. It wanted a model that would
—Assess the liquidity risk associated with its current credit-line commitments;
—Evaluate contingency and stress scenarios to estimate liquidity needs in extreme situations;
—Identify sources of potential risk exposure, such as concentrations in specific industries; and
—Evaluate the potential growth of its revolving-credit-line commitments and their use over three to five years for hypothetical portfolios.

The results of these analyses would directly affect management’s decision on liquidity risk and therefore the deployment of bank assets and the bank’s profitability.

The Modeling Approach

Modeling revolving credit facilities is complex largely because the borrowers can vary their usage. While some of the elements, such as rating transition probabilities are typically used in credit-risk models (Hirtle et al. 2001), we believe our work is the first comprehensive Monte-Carlo-based modeling of the liquidity behavior of a revolving-credit-facility portfolio that incorporates a Markovian rating-transition process, expert-system rules, and various degrees of correlation within and across industry sectors. Marker (1997) sketches a modeling methodology Citibank uses to value facilities rather than to evaluate the joint liquidity behavior of a portfolio of revolvers.

The underlying framework of our liquidity risk model is a Monte Carlo simulation (Figure 2). The model simulates monthly credit-line usage for each company and each tranche over a five-year (60-month) time horizon. During this time, credit ratings of companies change, tranches expire, some get renewed, some are terminated, and companies exercise term-out options. We took a Monte Carlo simulation approach because many of the key parameters are stochastic by nature. The simulation allows us to evaluate a wide range of scenarios and establish probability levels for the key outcomes the bank needs. The model incorporates the following OR/MS techniques:
—A Markov transition process to model the monthly stochastic changes in credit ratings for each company;
—The generation of correlated stochastic variables to capture correlations in credit-rating migrations arising from economic conditions that affect companies in a single industry or in different industries; and
—Expert-system business rules to determine when companies will exercise term-out options and whether the bank will renew or terminate expiring lines of credit.

The innovative combination of these OR/MS techniques to address the needs of ML Bank USA is what sets this work apart.

Stochastic variables in the model include:
—The monthly credit rating for each company;
—The probability that a company will start using a credit line;
—The probability that a company will continue using an already drawn credit line;
—The amount of the credit line used; and
—The sequence in which the company uses the different tranches.

We provide a process flow diagram of the model in Figure 2.

Monte Carlo Simulation

The simulation model projects the monthly liquidity requirements of the total existing portfolio of revolving credit facilities at Merrill Lynch by modeling each borrower’s timing and amount of credit use as a function of its credit rating and previous month’s use. Credit ratings are updated every month. A borrower rated \( a \) in month \( t - 1 \) will be rated \( a' \) in month \( t \) if the following holds:

\[
\sum_{b=1}^{a-1} p_{ab} < X \leq \sum_{b=1}^{a'} p_{ab},
\]

where \( X \) is the cumulative probability corresponding to a random draw of a normally distributed stochastic variable centered around rating \( a \), that is, the borrower’s rating in month \( t - 1 \). Credit migrations of borrowers are assumed to be correlated within and across industries.

A borrower’s probability of nonzero credit use in a given month is much higher if the borrower has
Figure 2: This process-flow diagram shows the simulation logic for each company for each month. The simulation processes are
A: Cumulative draw probability distribution (function of credit rating),
B: Draw probability conditional on currently using credit,
C: Distribution of percent drawn on available credit line (function of credit rating),
D: Credit rating transition matrix,
E: (Expert system) Term-out exercise rules (when a term out is exercised, usage is set to 100 percent for all tranches), and
F: (Expert system) ML tranche renewal rules.
drawn on its facility in the prior month than if not. The amount of use can vary randomly based on a user-defined distribution that takes into account a borrower’s right to draw deeper in its facility or prepay all or part of the loan balance without penalty.

We modeled default as an absorbing state, that is, it is impossible for borrowers to migrate out of the default status. The model assumes a borrower will increase use of its facility to 100 percent upon default and remain fully drawn throughout that particular simulation run. This is a conservative treatment of borrowers in default, because it implies total loss on a defaulted facility, even though expected loss is often less than 100 percent in reality, especially given the absolute priority of claims on bank loans over all other forms of the borrower’s debt and against its equity in the event of bankruptcy.

For borrowers that have facilities with term-out options, the business rules that govern whether they will invoke their term-out options depend on their credit rating the month the facility expires and the path it followed to get there. Because invoking a term-out option is a sign that the borrower’s credit quality is deteriorating, the bank usually refuses to renew the borrower’s credit line. Consequently, the model sets all the borrower’s facilities to full usage when the borrower exercises a term-out option on any one of them. The termed-out facility remains 100 percent drawn until the term-out period expires. At that time, it gets paid in full. The borrower’s other facilities remain 100 percent drawn until their expiration, at which time they get paid off. None of the affected facilities are renewed throughout that particular simulation run.

The bank reserves the right to not renew a facility of a borrower whose credit rating is below a certain threshold in the month the facility expires. The simulation assumes that a facility will be paid off if it is not renewed, except in the event that the borrower exercises a term-out option. If the borrower does not exercise a term-out option and has additional unexpired facilities, the bank may renew them if the borrower’s credit rating improves during subsequent months and is above the renewal threshold when they expire.

We developed the simulation model on version 7.01 of the Arena simulation software. A Visual Basic controller embedded in an Excel spreadsheet drives the model. The controller enables the user to input borrower data and revolving-credit-line data, to set parameters, to run the simulation, and to create reports. The simulation’s time horizon is typically 60 months, in line with the bank’s five-year planning horizon. To get reliable projections for the total liquidity needs of the portfolio over a multiyear time horizon, we tested the model extensively, and we determined that we needed at least 5,000 replications to obtain stable distribution results.

Random Number Generation and Industry Correlations

The random numbers assigned to stochastic variables in the model are generated by internal functions of the Arena software package. For most of the stochastic variables, we used the uniform distribution random number function in Arena, which generates a random value uniformly distributed between 0 and 1. We then compared the generated random number to the cumulative density function (specified as an input to the model) to classify the variable into a specific outcome. We used this approach for the following stochastic variables: credit-line use, amount of credit used, continued line use, and multiple tranche selection.

In the case of credit-rating transitions, we used a more complicated approach so that we could take industry correlations into account. That is, companies in the same industry are more likely to move up or down the credit-rating scale at the same time than companies in different industries. In this case, we need to generate stochastic variables that are correlated at a higher level for companies in the same industry and at a lower level for companies in different industries. We used a five-step decomposition process (Appendix). The decomposition process is complex and involves calculating eigenvalues, matrix transpositions, and square roots. But we can easily use the result to generate random values with the desired correlation properties. In Step 1, we generate a vector \( \mathbf{X} = [x_1, x_2, \ldots, x_C]^T \), where \( C \) is the number of companies with credit lines and \( X \) is a vector of independent, identically distributed standard normal stochastic variables (\( X \sim N(0, I) \), where \( I \) is the identity matrix). In Step 5, we obtain a vector \( \mathbf{Y} = \)
of correlated standard normal variables \( Y \sim N(0, R) \), where \( R \) is the industry correlation matrix) through special transformations of matrix \( R \) in Steps 2 through 4. We then compare the adjusted random numbers to the credit-rating transition matrix to determine the new credit rating for each company. An additional complication is that, to make this comparison, we need to transform the credit-rating transition matrix to a normally distributed cumulative density function. We do this in Excel, using the inverse normal distribution function (as a preprocessing step), which then feeds into Arena.

The model uses the SAS IML library of matrix algebra functions to decompose the industry-correlation matrix (Appendix). We developed the default correlations for companies in the same industry and in different industries through proprietary research as part of a separate study.

### The Credit-Migration Matrix

The credit-migration matrix is the most important input to the model because it drives the borrowers’ frequency and amount of facility use. The rows and columns of this square matrix correspond to credit ratings at months \( t-1 \) and \( t \), respectively. An element \( p_{ab} \) of the matrix represents the probability that a borrower with a credit rating of \( a \) in month \( t-1 \) will have a credit rating of \( b \) in month \( t \). Because they are probabilities, the elements of the matrix are non-negative and sum to 1 across rows. We assume that the credit-transition process is a discrete-time Markov chain, that a borrower’s credit rating in month \( t \) depends only on the rating in month \( t-1 \) and not on the path it followed to get there, and that the credit-migration matrix is constant throughout the simulation’s time horizon. Ross (1995) gives more information on Markov processes.

Credit ratings in the US are issued by four major agencies, of which the largest and best known are Standard and Poor’s (S&P) and Moody’s (Table 5), and the two smaller ones are Fitch, and Duff and Phelps. The credit rating measures the two major agencies use are somewhat different. S&P measures a borrower’s overall capacity to meet its debt obligation and hence its probability of default. Moody is believed to also incorporate in its ratings a judgment on borrowers’ expected recovery in the event of default, hence expected loss. Schuermann (2004) summarizes the extensive literature on expected loss, or loss given default.

A substantial body of work has accumulated on the measurement and estimation of credit-migration matrices (Jafry and Schuermann 2004). In our model, we use a one-month credit-migration matrix based on Moody’s ratings (Table 7). As customary, Caa includes all ratings below it (for example, Ca/CC, C and their subgrades) and above D, which represents a borrower in default.

### Expert System Rules

One of the unique components of the simulation model is the expert-system rules we incorporated to

<table>
<thead>
<tr>
<th>From/To</th>
<th>Aaa</th>
<th>Aa1</th>
<th>Aa2</th>
<th>Aa3</th>
<th>A1</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
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<th>Default</th>
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</thead>
<tbody>
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<td>95%</td>
<td>4%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>100%</td>
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</table>

Table 7: This sample transition matrix provides monthly credit-migration probabilities. The rows correspond to the current credit rating. The probabilities that an Aa2-rated borrower will be upgraded or downgraded one notch are three percent and four percent, respectively. Default is the absorbing state.
determine actions that Merrill Lynch and the borrower would take when specific events occur. These specific events are the bank’s decision to renew a credit line and the borrower’s decision to invoke a term-out option when credit lines expire.

Management Science and the ML Bank USA staff worked together to develop these business rules. We based the rules on industry practice and managerial judgment. They do not affect any of the stochastic variables used in the simulation model. The decision rules are a function of the credit rating of the borrower when the credit line expires and the number of downgrades in the borrower’s credit rating in the previous 12 months. We coded the expert-system rules into the Arena simulation, and the model invokes them each month for each company as needed.

We defined some parameters in the expert-system rules as variables that users can change. We designate these parameters in the rules below with (*) and show illustrative parameter values in italics.

The following rules (Figure 3) govern borrowers’ decisions to invoke the term-out option:

—When a term out is available, the company will exercise the option if

1. Its credit rating at expiration is CCC (*) or lower, AND
2. Its credit rating was lowered two (*) or more times during the previous 12 months.

—When companies exercise their term-out options, credit-line use is set to 100 percent across all tranches. The following rules (Figure 4) govern the decision by Merrill Lynch to renew a credit line when it expires:

—If a company is in default when a tranche expires, ML does not get paid and the line remains fully utilized.
—If a company exercises its term-out option, ML does not renew any tranches with that borrower when they expire.
—If a company does not exercise a term-out option, or it does not have a term-out option, then

1. If the credit rating is CCC (-) or lower, ML will not renew the line, and if the line is in use, assume it is paid off and the borrower is no longer in the portfolio;

OR

2. If the credit rating is higher than CCC (-), ML will renew the tranche.

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Figure 3: In this example of expert-system rules for term-out options, the decision to exercise a term-out option is a function of the borrower’s current (month t) credit rating and the steepness of the downgrade path to it. Below a certain rating threshold, term-out option exercises can be triggered by less steep downgrade paths.

Figure 4: In this example of expert system rules for credit line renewal, the decision to renew a credit line depends on a borrower’s current credit rating and whether it has exercised any term-out options. Below a certain threshold rating, term-out exercises are immaterial to the renewal decision.

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Implementation

In implementing the model, we made it easy and flexible for nontechnical users to operate. We installed run-time versions of the model on several workstations at ML Bank USA, where the staff runs it as needed. The model is driven through a Visual Basic controller embedded in an Excel spreadsheet. The controller allows the user to invoke all of the necessary steps to run the model and generate output. The process is simple and user friendly; it does not require knowledge of Arena or SAS. The system invokes and executes the software programs behind the scenes. The user enters data into the three cells in Step 1 and then clicks on each of the three remaining cells in sequence (Figure 5).

The input files required for the simulation model are extracted from the bank’s loan record database and processed into the formats Arena requires. During the input-file preparation (Step 2), the SAS IML matrix algebra function library is automatically called to decompose the industry correlation matrix and calculate array $B$. The input files passed to Arena include detailed facility data on each borrower (Tables 1 and 2), the rating-transition matrix, industry adjustments (array $B$), expert-system rules, and simulation controls. The user can change many of the parameters (for example, transition-matrix values, business-rule cutoffs and time limits, industry correlations, credit-use rules and probabilities) passed to the simulation to further customize the scenario.

In Step 3, the Visual Basic program invokes the Arena software to run the simulation. For a typical scenario, it takes approximately two hours to complete and generate the output files. Upon completion of the simulation, the Visual Basic program processes the model output into a user-friendly summary report in Step 4. The main outputs of the simulation are the following usage charts:

—The usage chart (Figure 6) shows projected liquidity requirements for the total portfolio over a 60-month time horizon. Usage at the 97.5th percentile and at the 99.95th percentile, which correspond to two and three standard deviations are the inputs to the bank’s planning process.

—The percent usage chart (Figure 7) shows the same statistics as the usage chart as a percent of the total commitment by month.

A wide range of additional outputs reflects the underlying richness of the simulation and helps users to analyze other projected characteristics of the revolver portfolio. For example,

—The usage-rating-default table (Table 8) tracks statistics on defaulted borrowers, such as the average, median, and percentiles of the number of borrowers in default and their credit use for each month in the simulation.

Another set of outputs shows average revolver utilization (Table 9) and number of borrowers (Table 10) for each credit rating across time.

Ongoing Improvements and Enhancements

The management science group supports and maintains the model and continually improves and enhances it in three main areas. We work with the bank to refine model parameters, such as expert-system rules and the tranche-usage sequence, by analyzing historical data on Merrill Lynch’s revolver portfolio. Jointly with the Moody’s rating agency, we update the credit-rating-migration matrix to incorporate the most recent rating-migration history. Finally, we work with Merrill Lynch’s risk-management group to obtain the most accurate industry-correlation estimates.

Business Impact

The liquidity risk model helps ML Bank USA manage the revolving credit lines. It provides a scientific and
Figure 6: In this projection of the five-year liquidity requirement of the revolving credit-line portfolio, revolving credit usage remains at or below the 97.5th percentile line in 975 out of 1,000 replications. Conversely, it exceeds the 97.5th percentile line in only 25 out of 1,000 replications. The 97.5th percentile line represents usage under “normal” conditions. The 99.95th percentile line is typically used in conjunction with extreme stress test scenarios.

Figure 7: In this graph of the projection of the five-year liquidity requirement of the revolving credit-line portfolio, we show credit line usage as a percent of the total portfolio commitment.
The bank uses the model to evaluate extreme-risk scenarios by changing model assumptions.

The bank uses the model in long-range planning. The bank continues to use the model monthly to evaluate liquidity and to manage its $13 billion portfolio of credit lines. To analyze basic liquidity, the bank typically runs the model once a month. The key outputs are the maximum credit-line use at the 99.95th and 97.5th percentile. The bank uses the vectors of monthly draw rates generated by the model at these percentiles as inputs into a separate stress-analysis model that it runs weekly. The results from the stress scenario analysis provide the bank with its overall stress liquidity measure.

Before ML Bank USA had the liquidity model, its assumptions about the liquidity reserves it needed for its revolving credit lines were very conservative. It assumed that borrowers could draw 50 percent of its outstanding credit commitments. Based on the model, this figure is now between 15 and 20 percent, a reduction of 30 percent or more. This change in its assumptions had the effect of releasing about $4 billion in additional liquidity that the bank can now deploy for other uses with higher returns.

ML Bank USA also uses the model to evaluate extreme risk scenarios, such as systemic financial-market problems that could affect the entire portfolio of borrowers. It evaluates such stress situations by changing all the industry correlations in the model to 1.0, greatly increasing the likelihood that companies will migrate towards lower credit ratings and potential defaults. Under this scenario, the model indicates that credit-line draws could double, but they would still be well below the old assumption of 50 percent of outstanding commitments.

The bank also uses the model to enhance its long-range planning. Semiannually the bank develops a four-year forecast of its balance sheet based on input from various business units. It uses the model results to project funded revolving credit facilities in the future. The projections improve its management of the balance sheet and the funding planning process. In addition, the bank occasionally uses the model to evaluate the impact of significant growth in revolving commitments over a three- or four-year time horizon. It does so by running the liquidity model with a hypothetical portfolio of increasing credit lines.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Month 1 (%)</th>
<th>Month 2 (%)</th>
<th>Month 3 (%)</th>
<th>...</th>
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<td>15</td>
<td>19</td>
<td>...</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 9: In this display of average revolver utilization for all borrowers in a credit rating, the boldface row indicates a sharp increase in utilization for borrowers rated Baa3 or lower, because access to less expensive funding sources is lost at those ratings.
Vector $Y$ will have covariance matrix $R$, as shown in the following proof:

$$\text{Var}(Y) = \text{Var}(BX) = \text{Var}(AEX) = (AE)\text{Var}(X)(E^T A^T) = A(EE^T)A^T = ADA^T = R.$$  

Note that $R$ must be a positive definite matrix.

### An Example of Industry Correlation Calculations

This example is for illustrative purposes, and we have changed the actual numbers.

#### Sample List of Companies

- Retail 1
- Retail 2
- Communications 1
- Communications 2
- Energy 1
- Energy 2
- Finance 1
- Finance 2
- Manufacturing 1
- Manufacturing 2

**Step 0.** Assume that correlation in the same industry is $r_1$; correlation in different industry is $r_2$ ($1 > r_1 > r_2 > 0$).

**Step 1.** Construct the correlation matrix for the above 10 companies:

$$R = \begin{bmatrix}
1 & r_1 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 \\
 r_1 & 1 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 \\
 r_2 & r_2 & 1 & r_1 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 \\
 r_2 & r_2 & r_2 & 1 & r_1 & r_2 & r_2 & r_2 & r_2 & r_2 \\
 r_2 & r_2 & r_2 & r_2 & 1 & r_1 & r_2 & r_2 & r_2 & r_2 \\
 r_2 & r_2 & r_2 & r_2 & r_2 & 1 & r_1 & r_2 & r_2 & r_2 \\
 r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & 1 & r_1 & r_2 & r_2 \\
 r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & 1 & r_1 & r_2 \\
 r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & 1 & r_1 \\
 r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & r_2 & 1
\end{bmatrix},$$

where $r_1 =$ correlation among companies in the same industry, and $r_2 =$ correlation among companies in different industries.

### Appendix

#### The Mathematical Process for Generating Correlated Random Variables

Problem: Generate random vector $Y$ with covariance matrix $R$, where $R$ reflects the correlations associated with companies in the same or different industries.

Solution: **Step 1.** Generate a vector $X$ of independent random variables from $N(0, 1)$, where $X = [x_1, x_2 \ldots x_n]^T$ and $C =$ number of companies with credit lines.

**Step 2.** Decompose $R$. $R = ADA^T$, where $D$ is a diagonal matrix of eigenvalues.

**Step 3.** Decompose $D$. $D = EE$, where $E = D^{1/2}$.

**Step 4.** Calculate $B = AE$.

**Step 5.** Calculate $Y = BX$, where $Y = [y_1, y_2 \ldots y_n]^T$ and is distributed as $N(O, R)$, and $C =$ number of companies with credit lines.

### Conclusion

The revolving-credit-liquidity model enabled ML Bank USA to improve its management of the liquidity risk associated with its $13$ billion revolving-credit portfolio and freed up $4$ billion of liquidity. The bank implemented and uses the model as part of its standard operating process. The model is a mix of several operations research techniques, including Monte Carlo simulation, Markov transition processes, and expert-system rules. In addition, it generates correlated stochastic variables to capture industry correlations in credit ratings. The model is an excellent example of the practice of management science yielding significant business benefits.
Step 2. Decompose $R$ into $R = ADA^T$, where $D$ is a diagonal matrix of eigenvalues.

$$A = \begin{bmatrix}
0.3162278 & -0.3133050 & 0.5087871 & -0.0495650 & -0.2012930 & 0.3536361 & 0.2052221 & -0.3286600 & -0.0235670 & 0.4735532 \\
0.3162278 & -0.3133050 & 0.5087871 & -0.0495650 & -0.2012930 & -0.3536360 & -0.2052220 & 0.3286600 & 0.0235666 & -0.4735530 \\
0.3162278 & 0.0889365 & -0.3255660 & -0.3869770 & -0.3692510 & -0.1130970 & 0.4541310 & 0.2868519 & 0.4324016 & 0.1082546 \\
0.3162278 & 0.0889365 & -0.3255660 & -0.3869770 & -0.3692510 & 0.1130966 & -0.4541310 & -0.2868520 & -0.4324020 & -0.1082550 \\
0.3162278 & -0.3172100 & -0.3348780 & 0.4310395 & 0.0379438 & -0.2648710 & -0.3850590 & -0.2201990 & 0.4229035 & 0.2328912 \\
0.3162278 & -0.3172100 & -0.3348780 & 0.4310395 & 0.0379438 & 0.2648706 & 0.3850591 & 0.2201986 & -0.4229030 & -0.2328910 \\
0.3162278 & 0.5415790 & 0.1516563 & 0.2873404 & -0.0335870 & 0.4825545 & -0.3056310 & 0.3666516 & 0.1948923 & 0.0362582 \\
0.3162278 & 0.5415790 & 0.1516563 & 0.2873404 & -0.0335870 & -0.4825540 & 0.3056306 & -0.3666520 & -0.1948920 & -0.0362580 \\
0.3162278 & 0.0000000 & 0.0000000 & -0.2818380 & 0.5661867 & 0.2431777 & 0.0998419 & -0.3560590 & 0.3092622 & -0.4565910 \\
0.3162278 & 0.0000000 & 0.0000000 & -0.2818380 & 0.5661867 & -0.2431780 & -0.0998420 & 0.3560586 & -0.3092620 & 0.4565910
\end{bmatrix}
$$

$$D = \begin{bmatrix}
3.77 & 0.87 & 0.87 & 0.87 & 0.55 & 0.55 & 0.55
\end{bmatrix}.
$$

Step 3. Construct $E = D^{1/2}$ as a diagonal matrix with values as the square root of the corresponding eigenvalues in $D$:

$$E = \begin{bmatrix}
1.9416488 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9327379 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.9327379 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.9327379 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.9327379 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.7416198 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.7416198 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7416198 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7416198 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7416198
\end{bmatrix}.
$$

Step 4. Construct matrix $B$ so that $B = A * E$:

$$B = \begin{bmatrix}
0.6140033 & -0.2223200 & 0.4745650 & -0.0462310 & -0.1877530 & 0.2622635 & 0.1521968 & -0.2437410 & -0.0174770 & 0.3511964 \\
0.6140033 & -0.2223200 & 0.4745650 & -0.0462310 & -0.1877530 & -0.2622640 & -0.1521970 & 0.2437408 & 0.0174775 & -0.3511960 \\
0.6140033 & 0.0829545 & -0.3066680 & -0.3609480 & -0.3444140 & -0.0838759 & 0.3367925 & 0.2127351 & 0.3206776 & 0.0802838 \\
0.6140033 & 0.0829545 & -0.3066680 & -0.3609480 & -0.3444140 & 0.0838747 & -0.3367930 & -0.2127350 & -0.3206780 & -0.0802840 \\
0.6140033 & -0.2587400 & -0.3123530 & 0.4020468 & 0.0353916 & -0.1964330 & -0.2855670 & -0.1633040 & 0.3136336 & 0.1727167 \\
0.6140033 & -0.2587400 & -0.3123530 & 0.4020468 & 0.0353916 & 0.1964333 & 0.2855674 & 0.1633037 & -0.3136340 & -0.1727170 \\
0.6140033 & 0.5051512 & 0.1414556 & 0.2680133 & -0.0313280 & 0.3578720 & -0.2266620 & 0.2719161 & 0.1445360 & 0.0268898 \\
0.6140033 & 0.5051512 & 0.1414556 & 0.2680133 & -0.0313280 & -0.3578720 & 0.2266617 & -0.2719160 & -0.1445360 & -0.0268900 \\
0.6140033 & 0.0000000 & 0.0000000 & -0.2628810 & 0.5281038 & 0.1803454 & 0.0744474 & -0.2640600 & 0.2293550 & -0.3386170 \\
0.6140033 & 0.0000000 & 0.0000000 & -0.2628810 & 0.5281038 & -0.1803450 & -0.0744450 & 0.2640601 & -0.2293550 & 0.3386170
\end{bmatrix}.$
Step 5. Generate the random independent vector $X$ from $\text{Normal}(0, I)$ with covariance $= I$ (identity matrix). Let $Y = B \ast X$. Then, $Y$ is a random sample that incorporates industry correlations (with covariance matrix $R$):

$$X = \begin{bmatrix}
0.23441 \\
-0.49978 \\
-0.17211 \\
-0.07346 \\
-0.62066 \\
1.03902 \\
0.54514 \\
0.94261 \\
0.89600 \\
-0.73138
\end{bmatrix},$$

$$Y = \begin{bmatrix}
0.1814252 \\
0.4750346 \\
0.4812961 \\
-0.747548 \\
-0.064951 \\
0.6530688 \\
0.9206000 \\
-0.130574 \\
0.2674681 \\
-0.596534
\end{bmatrix}.$$

Acknowledgments
Several people contributed to the success of this effort and deserve special recognition. We thank Andy Saperstein, chief operating officer and first vice president of Merrill Lynch’s financial advisory center, and Allen Braithwaite, first vice president of Merrill Lynch’s treasury, for their leadership and support throughout the project. We thank Bill Creager, director of marketing analytics, Gail Eisenkraft, first vice president of marketing strategy, and Paula Polito, senior vice president of GPC marketing, for their ongoing support of our analytical work. We extend our sincere thanks to Raj Nigam, retired chief scientist and director of the Merrill Lynch management science department for his leadership, friendship, and inspiration through many years. We thank Nenad Marinovich, head of credit-risk analytics at Merrill Lynch’s corporate risk-management group, for his help and contributions on the industry correlations. Last but not least, we thank the team in the Merrill Lynch analytics and management science department—Stuart Altschuler, Janet Chen, Mark Goldstein, Jukti Kalita, Gretchen Marsh-Ferino, Deepak Singh, Lonn Vreeland, and Zhaoping Wang—for their hard work and constant insistence on excellence.

References


Allen G. Braithwaite III, first vice president of ML & Co. Treasury, 4 World Financial Center, New York, NY 10080, made the following comments in a written memo dated August 23, 2004: “The revolving credit liquidity risk management model, which was developed and implemented in 2002, provided the first scientific measurement of liquidity risk in the Bank’s portfolio. The model increased Bank Treasury’s
confidence in measuring the liquidity required for the portfolio, which in turn resulted in freeing up approximately $4 billion of term liquidity. As management became comfortable with the model, the revolving line of credit portfolio increased substantially to about $13 billion. Over the last several years Management Science and Bank Treasury have collaborated on a number of innovative modeling projects, all of which are used regularly by Bank Treasury in measuring liquidity."