Closed book/notes/homework/cellphone quiz. Please write on one side of the paper only. Except for multiple choices, you will only receive full credit if you show all your work.

1. (7 points) Consider the transportation problem having the following parameter table:

After several iterations of the transportation simplex method, a BF solution is obtained that has the following basic variables: $x_{13} = 20$, $x_{21} = 25$, $x_{24} = 5$, $x_{32} = 25$, $x_{34} = 5$, $x_{42} = 0$, $x_{43} = 0$, $x_{45} = 20$. Continue the transportation simplex method for at most two iterations by hand. State whether the solution is optimal at each iterations, and if so, why?
Not optimal because $c_{41} - u_4 - v_1 = -4$, $x_{41}$ is the entering variable

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 5 & u_i \\
1 & 8 & 6 & 3 & 20 & 1 & 5 & 5 & 20 & 3 \\
2 & 5 & 4 & 7 & + & 25 & M & 8 & M-1 & 7 & 6 & 30 & 1 \\
3 & 6 & 3 & 9 & 6 & 5 & 8 & 5 & 30 & 3 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 \\
D & 25 & 25 & 20 & 10 & 20 \\
v_i & 4 & 0 & 0 & 3 & 0 \\
\end{array}
\]

$X_{42}$ is the leaving variable

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 5 & u_i \\
1 & 8 & 6 & 3 & 20 & 1 & 5 & 5 & 20 & 3 \\
2 & 5 & 4 & 7 & + & 25 & M & 8 & M-1 & 7 & 6 & 30 & 5 \\
3 & 6 & 3 & 9 & 6 & 5 & 8 & 1 & 30 & 7 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 \\
D & 25 & 25 & 20 & 10 & 20 \\
v_i & 0 & -4 & 0 & -1 & 0 \\
\end{array}
\]

Not optimal because $c_{31} - u_3 - v_1 = -1$, $x_{31}$ is the entering variable
$X_{34}$ is the leaving variable

Optimal, because $c_{ij} - u_i - v_j$ for all $x_{ij} \geq 0$
2. (4 points) Apply the Hungarian algorithm to solve the assignment problem having the following cost table:

<table>
<thead>
<tr>
<th></th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
<th>Task 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assignee</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>9</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Row reduction

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Row covering by 3 lines but there are 4 rows

Lowest non-zero $x_{ij} = 2$, now subtract 2 from non-covering cell and add 2 to double cover cell

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

now must use 4 lines to cover

A -> 1, B -> 2, C -> 4, D -> 3
3. Entenmann will be producing blueberry muffins at two different bakeries, and then the muffins must be shipped to two Schnuck’s supermarkets. Bakery 1 can send an unlimited amount by Entenmann trucks to Schnuck’s 1 only, whereas Bakery 2 can send an unlimited amount by Entenmann trucks to Schnuck’s 2 only. However, independent truckers can be used to ship up to 50 batches from each bakery to a distribution center, from which up to 50 batches can be shipped to each Schnuck’s. The shipping cost per unit for each alternative is

- Bakery 1 to Schnuck’s 1 = $7/batch
- Bakery 2 to Schnuck’s 2 = $9/batch
- Bakery 1 to distribution center $3/batch
- Bakery 2 to distribution center $4/batch
- Distribution center to Schnuck’s 1 $2/batch
- Distribution center to Schnuck’s 2 $4/batch

Allocation: Bakery 1 (80 batches) Bakery 2 (70 batches), Schnuck’s 1 (60 batches) Schnuck’s 2 (90 batches).

a. (2 points) Draw the network associated with this problem. Be sure you include all the constraints on the nodes and the arc. Be sure you show the netflow constraints in [ ] and also include the cost on the arc.

b. (2 points) Obviously, this is a minimum cost flow problem. Obtain an initial BF solution by solving the feasible spanning tree that corresponds to using just the Entenmann’s trucks plus bakery 1 shipping to Schnuck’s 2 via the distribution center. Be sure you use solid line to denote basic arc and use dashed line to denote nonbasic arc. Include flow in ( ).

c. (5 points) Perform network simplex problem for up to two iterations.
a. A: Bakery 1, B: Bakery 2, D: Schnuck's 1, E: Schnuck's 2, C: distribution center

\[ \Delta_{CD} = 2 - 7 + 3 = -2 \]
\[ \Delta_{BC} = 4 + 4 - 9 = -1 \]

\[ CD \text{ is the entering variable/arc} \]
\[ x_{CD} = \theta \leq 50 \]
$x_{AD} = 60 - \theta \geq 0, \theta \leq 60$

$x_{AC} = 20 + \theta \leq 50, \theta \leq 30$

AC is the leaving variable/arc, $\theta = 30$

$\Delta_{CA} = -3 + 7 - 2 = 2$

$\Delta_{BC} = 4 + 4 - 9 = -1$

BC is the entering variable/arc

$x_{BC} = 0 \leq 50$

$x_{BE} = 70 - \theta \geq 0, \theta \leq 70$

$x_{CE} = 20 + \theta \leq 50, \theta \leq 30$

CE is the leaving variable/arc, $\theta = 30$
\[ \Delta_{CA} = -3 + 7 - 2 = 2 \]
\[ \Delta_{EC} = -4 - 4 + 9 = 1 \]

The solution is optimal, giving the flow for the original diagram: