Closed book/notes/homework/cellphone quiz. Please write on one side of the paper only. Except for multiple choices, you will only receive full credit if you show all your work.

Please circle only one. Choose the best answer.

1. (1 point) Given an LP problem,
   a. the number of functional constraints is the same for both primal and dual problems.
   b. the number of decision variables is the same for both primal and dual problems.
   c. the sum of the number of functional constraints and the number of decision variables for both primal and dual problems are the same.
   d. All of the above are correct.
   e. Only (a) and (b) are correct.

2. (1 point) At each iteration, the Simplex method simultaneously identifies
   a. a CPF solution for the primal problem and a CPF solution for the dual problem.
   b. a BF solution for the primal problem and a BF solution for the dual problem.
   c. a BS for the primal problem and a BS for the dual problem.
   d. All of the above are correct.
   e. Only (a) and (b) are correct.

3. (1 point) If the primal problem has an unbounded objective function then the optimal value of the objective function for the dual problem
   a. must be zero.
   b. must not exist.
   c. must be unbounded.
   d. All of the above are correct.
   e. None of the above are correct.

4. (1 point) Regarding the Simplex method and primal and dual LP problems,
   a. a superoptimal BS in primal has a suboptimal complementary BS in dual.
   b. an optimal BS in dual has an optimal complementary BS in primal.
   c. at each Simplex iteration, a nonbasic variable in primal BS has a corresponding basic variable in complementary dual BS.
   d. All of the above are true
   e. Only (a) and (b) are true.
5. (8 points) Consider the following problem.

Maximize \( Z = 2x_1 - 2x_2 + 3x_3 \),

subject to

\[-x_1 + x_2 + x_3 \leq 4\]
\[2x_1 - x_2 + x_3 \leq 2\]
\[x_1 + x_2 + 3x_3 \leq 12\]

and \( x_1, x_2, x_3 \geq 0 \).

a. Construct dual problem for this primal problem.

b. Solve the primal problem with Simplex method in Tabular form. In each step

i. Identify the basic and nonbasic variables for the primal problem and the dual.

ii. Identify the basic solutions for the primal problem and the dual.

c. Identify the defining equations at the optimal solution for the dual problem.

Minimize \( W = 4y_1 + 2y_2 + 12y_3 \)

Subject to:

\[-y_1 + 2y_2 + y_3 \geq 2\]
\[y_1 - y_2 + y_3 \geq -2\]
\[y_1 + y_2 + 3y_3 \geq 3\]
\[y_1, y_2, y_3 \geq 0\]

\[
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
\hline
Z & -2 & 2 & -3 & 0 & 0 & 0 \\
x_4 & -1 & 1 & 1 & 1 & 0 & 0 \\
x_5 & 2 & -1 & 1 & 0 & 1 & 0 \\
x_6 & 1 & 1 & 3 & 0 & 0 & 1 \\
\end{array}
\]

RHS 0 4 2 12

Primal: Basic Solution \((x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 2, 0, 6)\), Basic variables \(x_4, x_5, x_6\) rest are nonbasic

Dual: BS \((y_1, y_2, y_3, z_1-c_1, z_2-c_2, z_3-c_3) = (0, 0, 0, -2, 2, -3)\), Basic variables are \(z_1-c_1, z_2-c_2, z_3-c_3\) rest are nonbasic

\[
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
\hline
Z & 4 & -1 & 0 & 0 & 3 & 0 \\
x_4 & -3 & 2 & 0 & 1 & -1 & 0 \\
x_3 & 2 & -1 & 1 & 0 & 1 & 0 \\
x_6 & -5 & 4 & 0 & 0 & -3 & 1 \\
\end{array}
\]

RHS 6 2 2 6

Primal: BS \((x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 2, 0, 6, 0)\), Basic variables \(x_3, x_4, x_6\) rest are nonbasic

Dual: BS \((y_1, y_2, y_3, z_1-c_1, z_2-c_2, z_3-c_3) = (0, 3, 0, 4, -1, 0)\), Basic variables are \(y_2, z_1-c_1, z_2-c_2\)
Primal: BS \((x_1, x_2, x_3, x_4, x_5, x_6) = (0, 1, 3, 0, 0, 2)\), Basic variables \(x_2, x_3, x_6\), rest are nonbasic

Dual: BS \((y_1, y_2, y_3, z_1-c_1, z_2-c_2, z_3-c_3) = (1/2, 5/2, 0, 5/2, 0, 0)\), Basic variables are \(y_1, y_2, z_1-c_1\)

The defining equations at \((y_1, y_2, y_3, z_1-c_1, z_2-c_2, z_3-c_3) = (1/2, 5/2, 0, 5/2, 0, 0)\) are:

\[
\begin{align*}
y_1 - y_2 + y_3 &= -2 \\
y_1 + y_2 + 3y_3 &= 3 \\
y_3 &= 0, z_2-c_2=0, z_3-c_3=0
\end{align*}
\]
6. (8 points) Consider the following problem.

Maximize \[ Z = x_1 - x_2 + 2x_3, \]
subject to
\[
\begin{align*}
2x_1 - 2x_2 + 3x_3 & \leq 5 \\
x_1 + x_2 - x_3 & \leq 3 \\
x_1 - x_2 + x_3 & \leq 2 \\
\end{align*}
\]
and
\[ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \]

Let \( x_4, x_5 \) and \( x_6 \) denote the slack variables for the respective constraints. After you apply the Simplex method, the final simplex tableau is as follows:

| Basic Variable \( \text{Eq.} \) | Coefficient of: \( \text{Right Side} \) |
|---|---|---|
| \( Z \) \( (0) \) | \( Z \) | \( 1 \) | \( 2 \) | \( 0 \) | \( 0 \) | \( 1 \) | \( 1 \) | \( 0 \) | \( 8 \) |
| \( x_2 \) \( (1) \) | \( x_2 \) | \( 0 \) | \( 5 \) | \( 1 \) | \( 0 \) | \( 1 \) | \( 3 \) | \( 0 \) | \( 14 \) |
| \( x_6 \) \( (2) \) | \( x_6 \) | \( 0 \) | \( 2 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 1 \) | \( 1 \) | \( 5 \) |
| \( x_3 \) \( (3) \) | \( x_3 \) | \( 0 \) | \( 4 \) | \( 0 \) | \( 1 \) | \( 1 \) | \( 2 \) | \( 0 \) | \( 11 \) |

You are now to conduct sensitivity analysis by independently investigating each of the following three changes in the original model. For each change, use the sensitivity analysis procedure to revise the given final set of equation (in tabular form) and convert it to proper form from Gaussian elimination, then test this solution for feasibility and for optimality. (Do not reoptimize).

a. Change the right-hand side of constraint 2 to \( b_2 = 4 \).

b. Change the coefficient of \( x_3 \) in the objective function to \( c_3 = 1 \).

c. Change the coefficient of \( x_1 \) in constraint 3 to \( a_{31} = 2 \).

a. \( b_{2\text{new}} = 4, \quad b_{\text{new}} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix} \)

\[ y^* = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \]

\[ y^* b_{\text{new}} = 9 \]

\[ B^{-1} b_{\text{new}} = \begin{bmatrix} 17 \\ 6 \\ 13 \end{bmatrix} \]
Feasible and optimal.

b. \( \mathbf{c}_{\text{new}} = [1 -1 1] \)

\[
\mathbf{z}^* \mathbf{c}_{\text{new}} = \mathbf{y}^* \mathbf{A} \mathbf{c}_{\text{new}} = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}
\]

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<tr>
<th>( x_1 )</th>
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<th>( x_3 )</th>
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Feasible and optimal

c. \( \mathbf{A}_{\text{new}} = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \mathbf{y}^* \mathbf{A}_{\text{new}} \mathbf{c} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \]

\[
\mathbf{B}^{-1} \mathbf{A}_{\text{new}} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 4 & 0 \end{bmatrix}
\]

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Feasible and optimal