AC POWER MEASUREMENTS

A. PREPARATION
   1. Review of Power and Power Factor
   2. Power in a Linear System Driven by a Periodic Voltage Waveform
   3. Power in a Nonlinear System Driven by a Sinusoidal Voltage Waveform
   4. The Measurement of Current and Voltage
   5. The Measurement of Single Phase Power and Power Factors
   6. References

B. EXPERIMENT
   1. Equipment List
   2. Safety Precautions
   3. Procedure
      a. Power Measurements for Resistive Loads
      b. The wattmeter
      c. Real and reactive power
      d. The nonlinear load
      e. Ground Wire Voltages
      f. DC waveform
   4. Report
   5. Addendum: Cord Set Convention
I. POWER MEASUREMENTS

A. PREPARATION

1. Review of Power and Power Factor

Consider the two terminal black box shown in Fig. 1.1.

A voltage $v(t)$ [V] across the terminals is associated with a balanced current $i(t)$ [A] entering at the upper terminal and exiting at the lower. The instantaneous power [W] is then defined to be

$$p(t) = v(t)i(t)$$  \hspace{1cm} (1.1)

while the average power [W] is defined to be

$$P_{\text{avg}} = \frac{1}{T} \int_0^T p(t)\,dt ,$$  \hspace{1cm} (1.2)
where, by convention, time zero is when we commence the measurement and $T \ [s]$ is long compared to any programmed variation in the supply. By convention, $P_{\text{avg}} > 0$ represents a sink for power and $P_{\text{avg}} < 0$ represents a source.

A classic example of these considerations is provided by

$$v(t) = \sqrt{2} |V| \cos(\omega t) \quad (1.3a)$$

$$i(t) = \sqrt{2} |I| \cos(\omega t - \theta), \quad (1.3b)$$

where $\theta > 0$ represents a lagging current and $\theta < 0$ represents a leading current; $\theta [\text{rad}]$ is called the phase angle. By Eq. (1.2),

$$P_{\text{avg}} = |V| |I| \cos \theta + |V| |I| \frac{\sin(2\omega T - \theta)}{2\omega T} + |V| |I| \frac{\sin \theta}{2\omega T}. \quad (1.4)$$

Clearly, as $T \to \infty$, the terms in $1/2\omega T$ vanish and Eq. (1.4) reduces to

$$P = \lim_{T \to \infty} P_{\text{avg}} = |V| |I| \cos \theta. \quad (1.5)$$

Further, since the mean square value of $v(t)$ is

$$<v^2(t)> = \lim_{T \to \infty} \frac{1}{T} \int_0^T v^2(t) \, dt = |V|^2, \quad (1.6)$$

$|V|$ is sometimes denoted by $V_{\text{rms}}$ (where "rms" means "root mean square") so that Eq. (1.5) can also be written as

$$P = V_{\text{rms}} I_{\text{rms}} \cos \theta. \quad (1.7)$$
It is not necessary to integrate to get from (1.3) to (1.7) if use is made of phasor techniques; moreover, by entering the frequency domain, further insight can be gained into the physical underpinnings of power flow. First, observe that

\[ v(t) = \text{RE} \left\{ \sqrt{2} |V(\omega)| e^{j\omega t} \right\} = \text{RE} \left\{ \sqrt{2}V(\omega) e^{j\omega t} \right\} \quad (1.8a) \]

\[ i(t) = \text{RE} \left\{ \sqrt{2} |I(\omega)| e^{j\omega t - j\theta} \right\} = \text{RE} \left\{ \sqrt{2}I(\omega) e^{j\omega t} \right\} ; \quad (1.8b) \]

obviously,

\[ V(\omega) = |V| \quad (1.9a) \]

\[ I(\omega) = |I| e^{-j\theta} . \quad (1.9b) \]

Then define

\[ S = P + jQ = V(\omega)I^*(\omega) = |V||I| \cos \theta + j|V||I| \sin \theta . \quad (1.9) \]

Here \( S \) (the complex power), \( P \) (the real power), and \( Q \) (the reactive power) would all appear to have the units of \([W]\); however, complex power is always specified in volt-amperes \([\text{V} \cdot \text{A}]\) and reactive power in volt-amperes reactive \([\text{VAR}]\). Finally, suppose that \( V(\omega) \) and \( I(\omega) \) are related by an impedance \( Z(\omega) \) such that

\[ \frac{V(\omega)}{I(\omega)} = Z(\omega) = R(\omega) + jX(\omega) , \quad (1.10) \]

where \( R(\omega) \) and \( X(\omega) \) are both real and have the units \([\Omega]\).

It then follows that
\[ I(\omega) = \frac{V(\omega)}{Z(\omega)} = \frac{|V|}{Z|e^{\text{arctan} X/R}}. \]  (1.11)

And from (1.9b) and (1.11)

\[ \theta = \text{arctan} \frac{X}{R}. \]  (1.12)

Two special cases exist:

(a) Capacitive Impedance. \( X < 0 \). \( \theta < 0 \).

Current leads voltage. \( \text{IM}[I(\omega)] > 0 \).

(b) Inductive Impedance. \( X > 0 \). \( \theta > 0 \).

Current lags voltage. \( \text{IM}[I(\omega)] < 0 \).

If now one refers back to Eq. (1.9) one sees that

\[ P = V_{\text{rms}}I_{\text{rms}} \cos \theta \]  (1.13a)

\[ Q = V_{\text{rms}}I_{\text{rms}} \sin \theta = P \tan \theta \]  (1.13b)

and that the sign of \( \theta \) matters only in setting the sign of \( \text{IM}[S] = Q \). The sign of \( P \) for a load is always positive because \( -\pi/2 \leq \text{arctan}(X/R) \leq \pi/2 \). \( \cos \theta \) is commonly termed the power factor and the following convention applied:

(a) leading power factor. \( \theta < 0 \) (capacitive).

(b) lagging power factor. \( \theta > 0 \) (inductive).

This convention is well worth committing to memory.

2. Power in a Linear System Driven by a Periodic Voltage Waveform

Suppose that for some reason the line voltage is nonsinusoidal and given by
\[ v(t) = \sqrt{2} \sum_{n=1}^{\infty} |V_n|\cos(n\omega t + \phi_n) . \quad (1.14a) \]

If the system is linear, it can be characterized by an impedance \( Z \) and hence by a current

\[ i(t) = \sqrt{2} \sum_{n=1}^{\infty} |I_n|\cos(n\omega t + \phi_n) . \quad (1.14b) \]

If now one applies Eqs. (1.2)-(1.5) and the addition formulas for the trigonometric functions, it follows that

\[ P = \sum_{n=1}^{\infty} |V_n| |I_n|\cos(\psi_n - \phi_n) . \quad (1.15) \]

That is, only currents and voltages of identical frequency will combine to yield a net power flow; the combination of a voltage and higher current harmonics yields no net power flow. Eq. (1.15) can be slightly better appreciated by reflecting that

\[ V_n^2 = |I_n|^2 e^{\phi_n} e^{\frac{j\psi_n}{2}} \]

\[ \Rightarrow \psi_n - \phi_n = \arctan \frac{X(n\omega)}{R(n\omega)} = \theta_n . \quad (1.16) \]

That is, \( \psi_n - \phi_n = \theta_n \) is the angle by which each current lags its voltage and is positive for inductive loads.

In short, Eq. (1.15) states that each frequency component is to have its power contribution evaluated separately; the individual contributions are then to be summed to get the total power.
3. Power in a Nonlinear System Driven by a Sinusoidal Voltage Waveform

While the power company strives to deliver a pure sinusoid, the user often insists that it drive a strongly nonlinear load such as a single phase rectifier or a fluorescent lamp. In these instances, the voltage fundamental at $\omega = 2\pi f$ can give rise in the current to a dc term and a sequence of harmonics:

$$v(t) = \sqrt{2} \, |V_1| \cos(\omega t)$$  \hspace{1cm} (1.17a)

$$i(t) = I_0 + \sqrt{2} \sum_{n=1}^{\infty} |I_n| \cos(n\omega t + \phi_n).$$  \hspace{1cm} (1.17b)

Clearly,

$$P = |V_1| |I_1| \cos(-\phi_1).$$  \hspace{1cm} (1.18)

However

$$I_{\text{rms}}^2 = |I_1|^2 + \left[ I_0^2 + \sum_{n=2}^{\infty} |I_n|^2 \right],$$  \hspace{1cm} (1.19)

so that a good deal more current may be flowing than one had anticipated.

The complex power in this instance is defined by

$$S_{\text{rms}}^2 = V_{\text{rms}}^2 \, I_{\text{rms}}^2$$

$$= P^2 + Q^2 + D^2$$  \hspace{1cm} (1.20)

where $P$ and $Q$ are the real and the reactive powers at power line frequency (cf. Eq. (1-9)) and
\[ D^2 = S_{\text{rms}}^2 - (P^2 + Q^2) = S_{\text{rms}}^2 - V_{\text{rms}}^2 |I_1|^2 \]

\[ = V_{\text{rms}}^2 \left\{ I_0^2 + \sum_{n=2}^{\infty} |I_n|^2 \right\}. \quad (1.21) \]

\( D \) is termed the distortion voltamperes.

Compensation to reduce \( Q \) and hence \( I_{\text{rms}} \) is quite possible by means of standard power factor adjusting techniques. Reduction techniques for \( D \) tend to be \textit{ad hoc}, and the reader is referred to the treatise by Shepherd and Zand (1979) for details. In general, a low distortion current is best achieved by good and thoughtful initial design of the system.

5. The Measurement of Single Phase Power and Power Factors

Remember the definitions of Eqs. (1.13) which can be expressed as
\[ P = V_{\text{rms}} I_{\text{rms}} \cos \theta \]  
\[ Q = P \tan \theta \]

where a lagging current (or power factor) is taken to imply 
\( \theta > 0 \) (inductive load).

The direct measurement of \( P \) is unusual in electrical engineering in that the measurement technique of choice today, the electrodynamometer wattmeter, was the technique of choice a century ago*. This instrument has a fixed coil carrying a current \( i_F(t) \) and a moving coil carrying a current \( i_M(t) \) and is so arranged that the instantaneous torque on the moving coil is proportional to the product \( i_F i_M \). If \( i_F i_M \) is constant (dc) or very rapidly varying compared to the natural frequency of the moving coil, the deflection of the moving coil will be proportional to the average torque. If then (as is the usual arrangement) the fixed coil is caused to carry the line current and the moving coil current is made proportional to the load voltage, the deflection will be a monotone function of \( P \) and a rugged, simple wattmeter will have been constructed.

* It is unclear what philosophical insights (if any) should be extracted from this state of affairs. But certainly it speaks well of the illustrious Ayrton and Perry who designed it. The student is of course familiar with these gentlemen since they coined the terms "ammeter" and "voltmeter" (Kinnard, 1962) and invented the celebrated Ayrton-Perry winding, much used in decade resistance boxes.
In modern electronic practice, the phase relationship between two periodic waveforms is most conveniently determined using a vector voltmeter of some sort. And in classical power practice it would have been measured using an arcane extension of the electrodynamometer principle (Kinnard, 1962). However, it can be simply (though not elegantly) measured by noting that 

\[ |S| = V_{\text{rms}}I_{\text{rms}} > P = |S|\cos\theta_L \]

and that the load admittance appears to be (see Fig. 1.5)

![Wattmeter Diagram](image)

\[ \text{Fig. 1.5} \]

Obviously (Let the reader so demonstrate!),

\[ G_L = \frac{P}{V_{\text{rms}}^2}, \quad (1.33a) \]

\[ \cos\theta_L = \frac{P}{|S|}, \quad (1.33b) \]

\[ |B_L| = G_L|\tan\theta_L|, \quad (1.33c) \]

\[ \text{sgn}B_L = -\text{sgn}\theta_L. \quad (1.33d) \]
What is not known is the sign of $\theta_L$ ($> 0$ for the usual inductive load). To determine $\text{sgn} \theta_L$, simply connect between terminals $T$ and $T'$ a capacitor of value $C_{sh} = \left| B_L \right| / \omega$. If $\theta_L > 0$, the inductive load will be compensated by $C_{sh}$ and the $P/|S|$ ratio will increase to approximately 1.0 (unit power factor). If $\theta_L < 0$, the capacitive load seen by the line will become more capacitive and the observed $P/|S|$ ratio will decrease. A still simpler test is to shunt the load with a big capacitor decade; if increasing $C_{sh}$ causes $I_{rms}$ to decrease, $\theta_L > 0$. And finally, the simplest technique for fixing $\text{sgn} \theta_L$ is to observe $\theta_L$ directly on the oscilloscope.
9. References


B. EXPERIMENT

1. Equipment List

1 instrument rack containing the usual lab gear
1 single-phase wattmeter
1 rheostat module with two 50-Ω rheostats rated at 4.5 Amp each.
1 10 A / 100 mV shunt
1 1000:1 current transformer
1 variable autotransformer (“Variac”)
1 custom-constructed Load Box

2. Safety Precautions

This course will probably be your first laboratory course in Electrical Engineering in which a significant opportunity for serious electrical shock exists. Previously you have worked with lower voltages and higher source impedances: here you will work with line power. This is not to say that the laboratory facilities themselves are dangerous, for they are not. Neither is it an assertion that the special equipment for the several experiments is dangerous; in fact, considerable effort has been spent to make it relatively foolproof. Rather, it is a statement that the student who adopts shoddy practices and dubious procedures is taking unnecessary chances and could conceivably sustain injury.

Therefore, Lab groups caught in miscellaneous minor violations of safety regulations will be fined one letter grade on the experiment in question. Lab groups caught in major violations will be flunked on the experiment in question.

The following safety regulations shall be in force at all times:

(i) The initial setup shall be made with the relevant panel outlets turned off and before the experiment is plugged into those outlets.

(ii) When a setup is to be reconfigured or rewired, it shall be turned off and disconnected from its power source. If DC voltages and capacitors have coexisted in the setup, the capacitors shall be shorted and discharged on short for at least 30 s.
(iii) No exposed power points shall be permitted to exist. In particular, the jerry-rigging of extension cables by plugging together short jumpers is strictly forbidden.

(iv) All metallic chassis shall be grounded. Check for a green banana receptacle.

(v) When shifting sensor probes in a hot circuit, be sure to shift using one hand only: keep the other behind your back.

(vi) Do not crowd around the apparatus. Do not jostle one another.

(vii) The initial setup and any reconfiguration shall be checked by the instructor before any power is applied to the circuit.

(viii) Loose garments, dangling neckties, sweat shirt ties, and long hair can become snarled with the shafts of rotating machines. Beware!

3. Procedures

a. Load Resistance Measurements.

   Use a DMM as an ohmmeter to set each of the pair of rheostats to 25 Ω. It is usually best to start with both rheostats set to mid-range thereby avoiding the possibility of a short circuit when the circuit is energized. Note that the rheostats are rated at 4.5 Amp and that the Load Box fuse is rated at 7 Amp. Then connect the rheostats in parallel to give a 12.5 Ω load capable of 9 Amps. Now use a DC power supply to make an in circuit four-terminal measurement of the effective parallel load resistance. Be sure to take careful notes of your methodology.

   Now connect the Load Box to the single-phase AC wall outlet via a Variac and connect the variable parallel rheostat load to the load box HI/LO output terminals. Use both a Current Transformer and a Current Shunt with the two rack-mounted DMMs to obtain two independent measurements of the load current. Use an auxiliary DMM to measure the output voltage of the Load Box. Be sure that the Load Box switches are set to "Linear" and "X = 0". Increase the Variac voltage slowly from 0% and take measurements at load currents of 2, 4, and 6 Amp using the shunt as the reference. Assuming that the rheostats are purely resistive, calculate the effective parallel load resistance for each measurement.
b. **Power Measurement for Resistive Loads.**

Now reconfigure the above setup to contain an appropriately inserted wattmeter of at least 1 kW rating. Increase each rheostat to maximum resistance in order to provide a load resistance of approximately 24 Ω. Drive the apparatus at 60 Hz with a Variac. Vary the input voltage (Terminals 1 & 3) and obtain data illustrating the variation of \(|S|\) with \(P\) from \(P = 100\) W to \(P = 500\) W in steps of 100 W. Use both a 1000:1 current transformer and a current shunt to measure the current§.

c. **Reactive power.** Alter the setup of Part b. above by flipping the appropriate switch to "\(X \neq 0\)" and adjust the rheostats to obtain a resistance of about 12 Ω. Vary the Variac voltage up to 120 Vrms and obtain a data set of input voltage (V) (Terminals 1 & 3) and shunt current (I) versus power power (P). ♣ Using the oscilloscope determine both the magnitude and sign of the power factor angle \(\theta_L\) for an rms input voltage of about 60 V. Measure P, V, and I at this condition and make a hardcopy of the scope display. Finally, when finished using the Load Box, measure its inductance and resistance using the LCR meter.

d. **The nonlinear load.** Reset the appropriate switch to "\(X = 0\)" and recreate an approximately 25-Ω load capable of handling more than 4.5 A. Set the input voltage to 120 V\(_{\text{rms}}\) and maintain it there. Carefully observe the input and load voltage waveforms. Now flip the appropriate switch to the "**NONLINEAR**" position and measure in P, V, and I and observe the significant shifts* in input and load voltage waveforms. Make a hard copy of these waveforms.

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§ Observe that the manufacturer may not guarantee the accuracy of his device at low currents. Throughout this course, you will have to consider the design tradeoffs of using a Hall probe versus using a transformer versus using a current shunt versus using an ammeter directly.

♣ Increasing the voltage will rapidly push the current above the rated continuous maximum of the rheostats: 4.5 A. Clearly, this calls upon you to devise a strategy to overcome this unforeseen difficulty. Be sure to document your solution to the problem.

* This involves an element of design in that you cannot take note of everything, are therefore **constrained** to select salient features, and are striving to **minimize a cost function** (demerits assigned by the grader).
e. **Ground wire voltages.** There are four (4) grounded wires available to the experimenter in this laboratory:

1. The standard green grounding conductor of the ordinary 120 VAC circuits. This is really close to Mother Earth potential of zero voltage unless of course it is (i) carrying a heavy current due to a fault condition or (ii) something has been wired wrong.

2. The standard white neutral wire of the ordinary 120 VAC circuits. This is solidly grounded *somewhere* but normally carries current and gets floated a short distance away from Mother Earth potential by the resultant IR drop.

3. The green neutral of the DC supply. This is well and thoroughly grounded and will be floated away from Mother Earth potential only if someone either (i) has connected a single-sided load improperly or (ii) is operating with an unbalanced two-sided load.

4. The "green" neutral of the 3-φ supply. This too is well and thoroughly grounded and will remain at Mother Earth potential as long as all 3-φ loads are balanced and no neutral current exists to produce an IR float.

*Not even the green grounding conductor is absolutely trustworthy!* Nevertheless, the reason that there are *green grounding posts* on most chassis is to enable grounding to the green ground. **ALWAYS KEEP YOUR CHASSIS GROUNDED.**

Use the scope to observe the voltage of the white neutral wire of the laboratory AC power source with respect to the green grounding wire as described herein. Vary the load conditions using the Load Box connected directly to the 120 VAC wall outlet. Be sure that its switches are set to "**Linear**" and "**X = 0**". Observe the waveform between point "3" of the Load Box and the chassis ground on the scope and record the peak-to-peak voltages for the following four cases: (i) no load (i.e., open circuit); (ii) 2 Amp load; (iii) 4 Amp load; (iv) 6 Amp load. Be careful not to exceed the 4.5 Amp rating of the rheostats. Capture the display for each of these cases.

f. **DC supply.** Using the Load Box connected to the DC source via the DC cord, examine the voltage waveform of the DC wall outlet for the no load case. Make a hard copy and determine its dc level (i.e., average value), its peak-peak ripple amplitude, and the frequency of its ripple.
4. **Report**

In the report, one ought always strive to be laconic, legible, and lucid. The objective is to demonstrate your ability to gather data correctly, to analyze data appropriately, and to comprehend its meaning completely. The instructors are interested in neither "boiler plate" nor meretricious piffle.

**NOTA BENE:** The approved graphing technique consists of (1) obtaining a suitable graph sheet or graphing program*, (2) drawing a smooth theoretical curve (if applicable), and (3) inserting easily distinguished experimental points. Please avoid (i) graphs of many colors, (ii) graphs with experimental curves (unless the experiment truly did yield a continuous curve), and (3) excessively busy graphs.

**NOTE:** Letters of the following paragraphs relate to steps in the procedure.

a. Tabulate the DC and AC four terminal effective parallel load resistances you calculated based on shunt currents. What is the average resistance? Now plot the 4 resistances versus shunt current. How well do these resistances compare?

b. Assuming that the two rheostats are purely resistive elements*, tabulate (with measured AC power as the independent variable) the measured AC input voltage, AC shunt current, experimental volt-amperes, and experimental $I^2R$. Are the calculated results reasonable?

Plot the Current Transformer AC current versus the Current Shunt AC current. Obtain a linear regression line and show it on the data plot. Comment on the advantages and disadvantages of the two methods of measuring AC currents*.

c. Assuming the two rheostats to be purely resistive elements, tabulate measured input power, load voltage, shunt current, experimental volt-amperes, power factor

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* Logarithmic or other linearizing transformations may prove exceedingly useful in such endeavors.
* The "A" student will, naturally enough, have thought to measure the inductance.
* Kindly refrain from vague, ambiguous, meaningless, or without-added-value statements related to what was already obvious from the data themselves.
and inferred phase angle. Show the equations used to calculate the inductance. Plot calculated inductance versus measured input voltage and compute a best estimate of $L$. How does this value of $L$ compare with your direct measurement of $L$ using the LCR meter?

On a single sheet, display a plot your $P$ versus $|S|$ data and compute a $\theta_L$; succinctly explain your method. Clearly describe the procedure you designed* to determine the value of $\theta_L$ using the oscilloscope. What were the values of $\theta_L$ in the above two cases and were these values in satisfactory agreement?

How can the +/- signs on the wattmeter be used to ensure an upscale indication?

d. In unambiguous tabular form present your data on $V$, $I$, $|S|$ and $P$. Present a hard copy of the input voltage and nonlinear load voltage waveforms. How might power factor sensibly be defined for a non-linear load? Provide an equation for power factor and calculate its value.

e. Tabulate the peak-peak neutral wire voltages measured in your ground wire voltage study? Does this data reveal anything useful to you?

f. What was the average DC wall outlet voltage? What were the amplitude and frequency of the ripple on the DC wall outlet? Assuming that the DC voltage is provided by unfiltered, three-phase, full-wave rectification, what ripple frequency and peak-to-peak amplitude would be expected and why?\hspace{1em}^\ddagger

5. Addendum

Cord Set Convention

The following three sheets show the wiring conventions for the cord sets used in the several experiments of this course. It is obvious that the Jones plugs (Cinch S0404 CCT) on the chassis can be connected to more than one cord set. Take care in what you are doing lest you blow out a chassis!

* If the grader can’t readily envision your setup, the description is NOT clear enough.
\hspace{1em}^\ddagger Be informed that, from long experience, the instructor will be checking your explanation with some care.