Problem 1.- Compare ROTH-IRA to Traditional IRA

$P = \$2,000$

Option 1. ROTH-IRA Pay taxes now

$\$2,000 \times 0.3 = \$600$

IRA account $= \$1,400 = P$

$i_{eff} = 0.08$

$n = 70 - 22 = 48$ years

$i_{eff} = \left(1 + \frac{i}{K}\right)^K - 1$

Assume interest compounded yearly $\rightarrow i = i_{eff}$

$\boxed{F} = P(1+i_{eff})^n = \$1,400\,(1+0.08)^{48} = \boxed{\$56,895}$

Option 2

$P = \$2,000$

$i_{eff} = i = 0.08$

$n = 48$

$F = P(1+i_{eff})^n = \$2,000\,(1+0.08)^{48} = \$80,421$

Taxes $= 0.35 \times \$80,421 = \$28,147$

$\boxed{\text{Money available}} = \$80,421 - \$28,147 = \boxed{\$52,274}$

Based on this analysis, it is in your best interest to choose the Roth-IRA if you anticipate you'll belong in a higher tax bracket when you retire.

(Problem 1 - Continued)

Option B:

$$\begin{cases} P = \$2{,}000 \\ Tax = 0.3 \end{cases}$$

Tax initial on $P = 0.3 \times \$2{,}000 = \$600$

$P_{real} = \$1{,}400$

Assume the bank pays $i_{eff} = 0.08$ to you too

1st year $\quad F_1 = P + Pi - \overbrace{0.3 \times Pi}^{TAX} = P(1+0.7i)$

$$F_2 = P(1+0.7i) + P(1+0.7i)i - \overbrace{0.3P(1+0.7i)i}^{TAX}$$

$$= P(1+0.7i)(1+0.7i) = P(1+0.7i)^2$$

$$\vdots$$

$$F_{48} = P(1+0.7i)^{48} = 1{,}400(1+0.7(0.08))^{48}$$

$\boxed{F_{48} = \$19{,}142}$ Amount in your account at age 70.

Problem 2

First calculate after tax earnings

Let S_n be amount in bank at year n.

At end of year, interest earned $= S_n \cdot i$

Taxes paid $= S_n \cdot i \cdot t$

where t = tax rate

Account balance, $S_{n+1} = S_n + S_n i (1-t)$

$$= S_n [1 + i(1-t)]$$

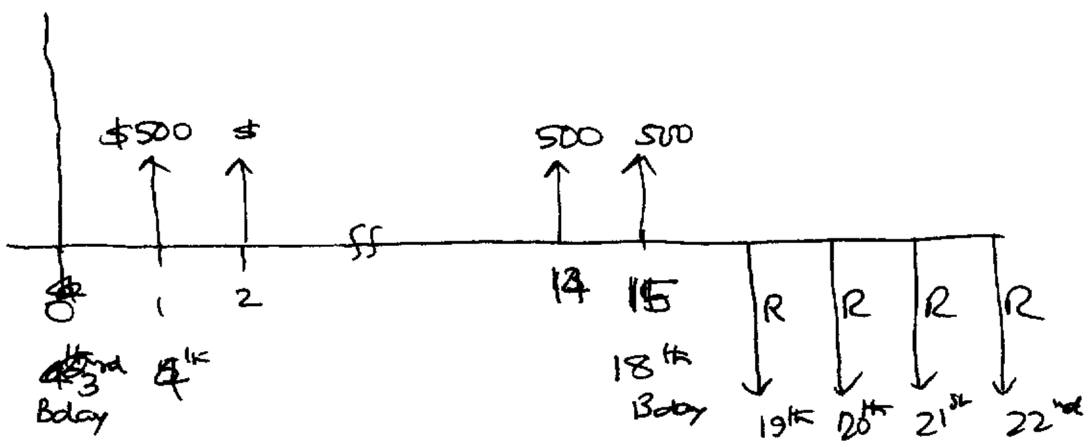
So taxes reduce the interest paid

$i = 0.10$

$t = 0.30$

$$\therefore i_{eff} = i(1-t) = .10(1-.30)$$
$$= .07 \text{ or } \underline{7\%}$$

Cash flow diagram



Account Balance on ~~15th~~ 18th Bday

FW of an Annuity ~~R~~

$$FW = R\left[\frac{(1+i_{eff})^n - 1}{i_{eff}}\right]$$

$$= \$500\left[\frac{(1+.07)^{15} - 1}{0.07}\right]$$

$$\boxed{FW = \$12564.51}$$

Withdraws, 4 equal inst. use PW formula

$$PW = R\left[\frac{(1+i)^n - 1}{(1+i)^n \cdot i}\right]$$

$$\$12564.51 = R\left[\frac{(1+.07)^4 - 1}{(1.07)^4 \cdot .07}\right]$$

$$\boxed{R = \$3709.41/\text{year}}$$

With an Education IRA:

$$i = 0.10$$

$$\therefore FW = \$500\left[\frac{(1+.10)^{15} - 1}{.10}\right]$$

$$= \$15886.24$$

$$\boxed{R = \$5011.31/\text{yr}}$$

This is 25% larger. Hence use this option.

Problem 3: (Textbook # 9)

Given: A \$500,000 investment and the expected annual cash flows over six years (project life)

Find: Whether it's better to go for a fixed 6% p.a. effective interest rate in a bank or not.

Solution:

Year	Annual Cash Flow
1	\$25,000
2	\$50,000
3	\$150,000
4	\$250,000
5	\$100,000
6	\$75,000

Bank option: $F = P(1+i)^n = \$500,000(1+0.06)^6 = \$709,260$

Investment option:

1st year = \$25,000

2nd year = $\$25,000(1.06) + \$50,000 =$

3rd year = $\$25,000(1.06)^2 + \$50,000(1.06) + 150,000$

⋮

6th year = $\$25,000(1.06)^5 + \$50,000(1.06)^4 + \$150,000(1.06)^3 + \$250,000(1.06)^2 + \$100,000(1.06) + \$75,000$

6th year = \$737,132

Since \$737,132 > \$709,260 The company should invest the money in the process improvement process.

Problem 4 (Text. Page 133 - #15)

Given: A new \$100 million plant paid in two installments that generates annual cash flows of \$25 million after startup

Find: a) Draw discrete and cumulative CFD for the 17 year investment

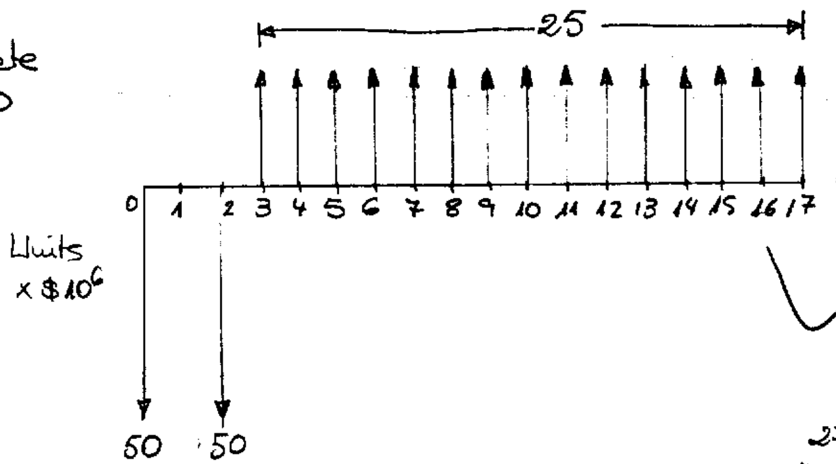
b) Future value of project at the end of year 17.

c) Equivalent value of this investment in today's dollars (cash flows to zero)

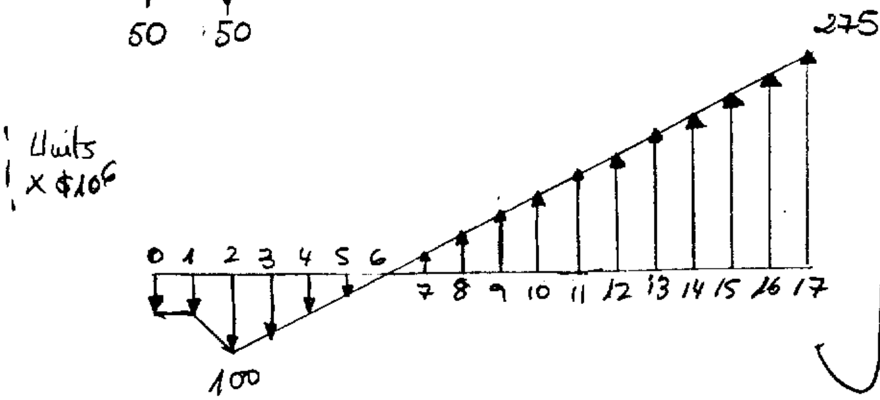
d) What ieff would make the investment in today's dollars be zero?

Solution:

a) Discrete CFD



Cumulative CFD



b) $i_{eff} = 0.07$

$$F = (-\$50)(1+0.07)^{17} + (-50)(1+0.07)^{15} + (\$25)(F/A, 0.07, 15)$$

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i} = \frac{(1+0.07)^{15} - 1}{0.07} = 25.13$$

$$\boxed{F = -157.94 - 137.95 + (\$25)(25.13) = \$332.36 \text{ million}}$$

Future value at the end of year 17th.

c) $i_{eff} = 0.07$

Exponents become negative

$$P = -\$50 + \frac{-50}{(1.07)^2} + \left[\underbrace{\frac{25}{(1.07)^3} + \frac{25}{(1.07)^4} + \cdots}_{(\$25)(P/A, 0.07, 15)}\right](1.07)^{-2}$$

Alternatively: we know the future value at time 17, bring that total back to year zero.

$$P = (\$332.36)(1+i)^{-17} = \$332.36(1.07)^{-17}$$

$$\boxed{P = \$105.22 \text{ million}}$$

Or using the annuity formula:

$$P = -\$50 + \frac{-\$50}{(1.07)^2} + (\$25)(P/A, 0.07, 15) \times (1.07)^{-2}$$

$$P = -\$50 - \$43.67 + (\$25)(9.1079)(1.07)^{-2}$$

$$\boxed{P = \$105.21 \text{ million}}$$

d) $$P = -\$50 - \frac{\$50}{(1+i)^2} + 25\frac{(1+i)^{15} - 1}{i(1+i)^{15}} \times (1+i)^{-2} = 0$$

$$-\$50\, i(1+i)^{17} - \$50\, i(1+i)^{15} + \$25(1+i)^{15} - 25 = 0$$

$\boxed{i = 0.192}$ (See next page)

Can Also solve by plotting P vs. i