Solution to PS#11: Optimization

Problem 1

pumping costs = $k_1 U$ \$/year = C_o

cost of pipe = $k_2 D + k_3$ \$ = C_c

Annualized capital cost factor = r = depreciation + maintenance + interest charges
= 0.10 + 0.05 + 0.10
= 0.25

velocity $U = \dfrac{q \text{ ft}^3/\text{sec}}{\pi D^2/4 \text{ ft}^2} = \dfrac{4q}{\pi D^2}$ ft/sec

Venture cost = $C_o + r C_c$
= $k_1 U + r.(k_2 D + k_3)$ \$/yr

$C_T = k_1 \left(\dfrac{4q}{\pi D^2}\right) + 0.25 (k_2 D + k_3)$

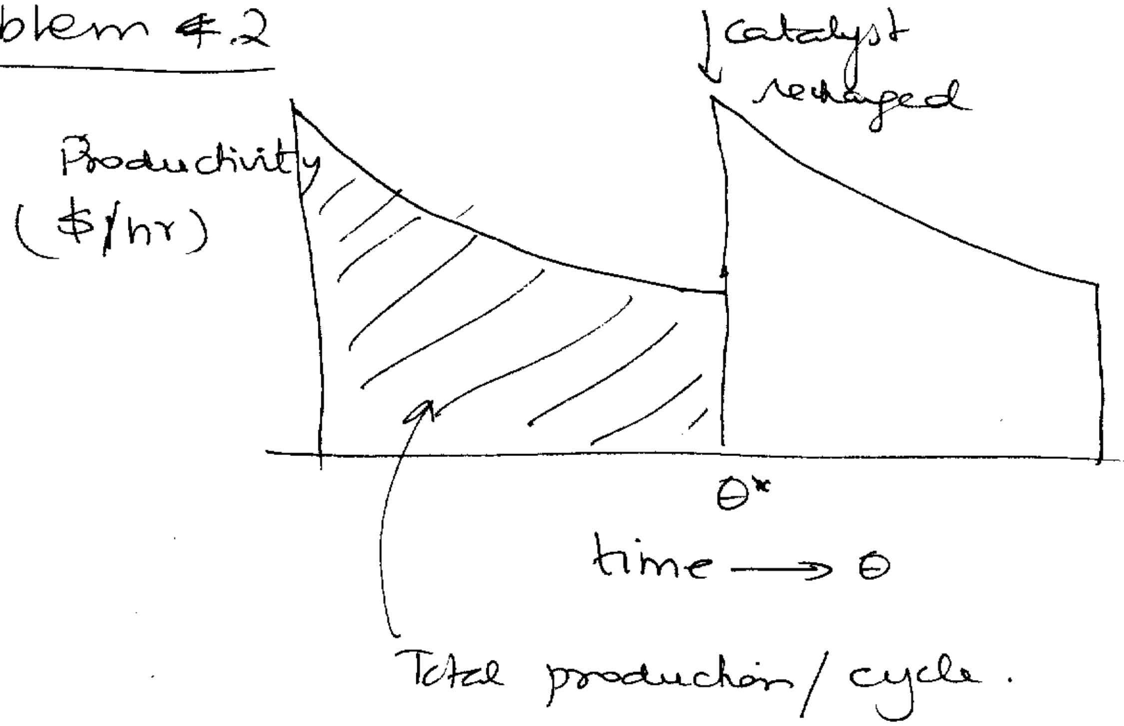
At optimum diameter

$$\frac{dC_T}{dD} = 0$$

$$\frac{dC_T}{dt} = \frac{-4 k_1 q}{\pi D^3}(2) + 0.25 k_2$$

or $$D^{*3} = \frac{8 q k_1}{0.25 \pi k_2}$$ $$\therefore D^* = \sqrt[3]{\frac{32 q k_1}{\pi k_2}}$$

Problem 4.2



Net Production in one cycle

$$= \int_0^{\theta^*} P\,d\theta \quad , \text{ where } \quad P = \$/hr$$

$$= \int_0^{\theta^*} \left(P_0 e^{-a\theta}\, d\theta\right)$$

$$= \frac{P_0}{a} - \frac{P_0}{a} e^{-a\theta^*}$$

Recharging cost = \$ C \$

~~Total cost~~ ~~Net~~ Profit/ $p = \left(\frac{P_0}{a} - \frac{P_0}{a} e^{-a\theta^*} - c\right)$ \$/cycle

Want to maximize Profit/hr or profit/year.

Profit $P_h(\$/hr) = \dfrac{P}{\theta^*}$

$$= \frac{P_o}{a\theta^*} - \frac{P_o}{a\theta^*} e^{-a\theta^*} - \frac{c}{\theta^*}$$

$\therefore \dfrac{dP_h}{d\theta^*} = 0$ at optimum

$$\frac{dP}{d\theta^*} = \frac{-P_o}{a\theta_*^2} + \frac{P_o}{a\theta^{*2}} e^{-a\theta^*} + \frac{P_o}{a\theta^*}(a) e^{-a\theta^*}$$

$$+ \frac{c}{\theta^{*2}} = 0$$

$$\therefore -\frac{P_o}{a} + \frac{P_o}{a} e^{-a\theta^*} + P_o\theta^* e^{-a\theta^*} + c = 0$$

$$\therefore \boxed{\frac{c}{P_o} = \frac{1}{a} - \frac{1}{a} e^{-a\theta^*} - \theta^* e^{-a\theta^*}}$$

This is a nonlinear equation for θ^*.
Must solve for θ^* by trial and error.

Problem 3 Solution

Let x_A = A used

x_B = B "

x_C = C "

$$\text{Cost} = 1.40 x_A + 2.00 x_B + 3.00 x_C$$

$$x_A + x_B + x_C = 10 \Rightarrow x_C = 10 - x_A - x_B$$

$$\therefore \text{cost} = ~~30~~ \; 1.40 x_A + 2.00 x_B + 3.00(10 - x_A - x_B)$$

$$= 30 - 1.60 x_A - 1.00 x_B$$

Constraints:

Mango: $0.1 x_A + 0.1 x_B + .0.4(10 - x_A - x_B) \geq 10(0.25)$

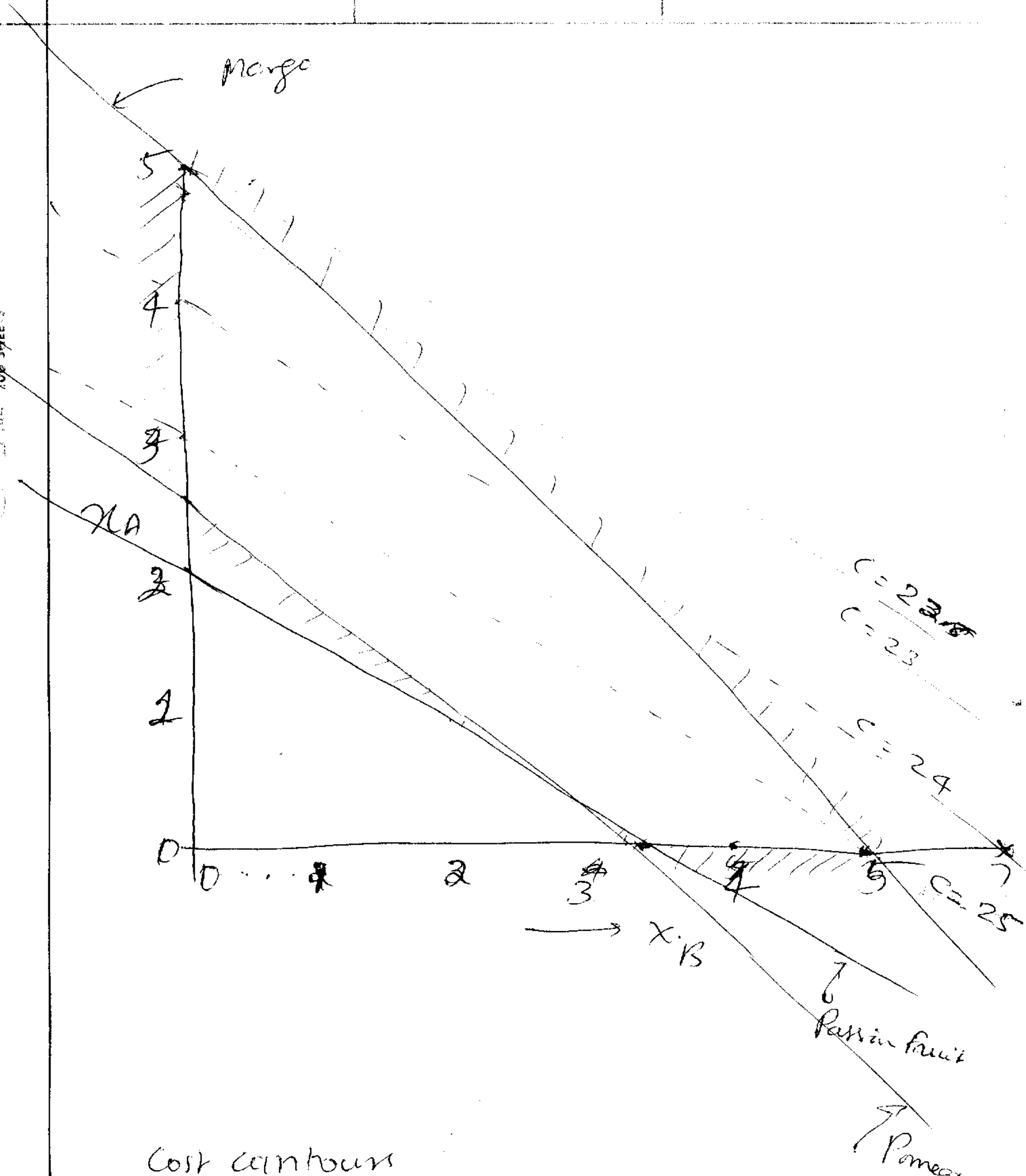
$$\therefore .3 x_A + .3 x_B \leq 1.5$$

Passion Fruit: $.1 x_A + .3 x_B + .6(10 - x_A - x_B) \leq 10(.50)$

$$.5 x_A + .3 x_B \geq 1.0$$

Pomegranate: $.8 x_A + .6 x_B \geq 10(.2) = 2.0$

Also $x_C \geq 0 \Rightarrow 10 - x_A + x_B \geq 0 \quad x_A + x_B \leq 10$



Cost contours

$$C = 30 - 1.6x_A - 1.00x_B = \sout{40}\ \sout{20}\ 25$$

$$30 - 1.6x_A - 1.00x_B = 24$$

$$30 - 1.6x_A - 1.00x_B = 23$$

Minimum cost at $x_A = 5,\ x_B = 0,\ x_c = 5$

$$C = 30 - 1.6 \times 5 = \underline{\underline{22}}$$