Fundamental Principles of Heat Transfer

Heat is energy in transfer due to a temperature difference. The three basic mechanisms of heat transfer are conduction, convection and radiation.

For our lab Experiment 2, we will only consider conduction and convection.

**Heat Conduction** - Conduction is the basic mechanism for heat transfer in solids. In a molecular scale, it is modeled by a transfer of vibration energy of adjacent fixed atoms from those of high vibration energy (temperature) to those of lower vibration energy. The fundamental equation is Fourier's law of heat transfer, which is the counterpart of Newton's law of viscosity

where:

\[
\frac{q}{A} = -k \frac{dT}{dy}
\]

A - area of heat transfer surface
y - distance normal to the surface
q - rate of heat flow across the surface
dT/dy - Temperature gradient in the direction normal to the surface
k - thermal conductivity of the material

The thermal conductivity is a property that measures the ability of the material to conduct the heat. The higher the thermal conductivity, the higher the heat flow per unit area at a given temperature gradient.

Let us consider a flat slab of solid with a temperature gradient:

At steady state, the heat flow at every point in the direction of flow is the same. Since the area is the same:
The thermal conductivity varies slightly with temperature and can be considered constant over small temperature ranges.
For wide temperature ranges, the thermal conductivity may be assumed to vary linearly with temperature.

Heat Convection
The molecular model of heat transfer by convection is based on the mobility of fluid molecules. Molecules acquire internal energy from the hot surface and travel to collide with a less energetic molecule with which it then shares its energy. Thus energy is transferred from the hot surface to the colder one. This mobility makes the heat flow through a liquid to be higher than that of a solid of equal thermal conductivity for the same temperature gradient. The heat flow increases even more when the fluid flows past the surface either by natural convection; the gravitational force due to the density gradients created by the temperature gradient itself forces the convection.

Film Coefficient - Let us consider the case of a fluid moving past a hot surface:

If the fluid is moving in turbulent flow, the bulk of the fluid will be maintained at a uniform temperature by the mixing of the turbulent eddies. However there is a viscous sub layer near the surface where the flow is laminar. In the film model of convection heat transfer, the resistance to heat flow is assumed to be the conduction resistance of the thin viscous film near the wall:
\[ q = Ak \frac{T_w - T_b}{\delta} = hA(T_w - T_b) \]

\( k \) = thermal conductivity of the fluid  
\( \delta \) = thickness of the viscous sub layer (unknown)  
\( h \) = film coefficient of heat transfer  
\( T_w \) = temperature at the surface  
\( T_b \) = temperature of the bulk of the fluid

It can be inferred from the equation that the film coefficient of heat transfer will be a function of the thermal conductivity of the fluid and of the Reynolds number and therefore the fluid flow rate since Reynolds number is directly proportional to the flow rate. The higher the flow rate, the lower the sub layer.

This is why it is important from a control viewpoint to design a heat transfer system that has a high film coefficient. It improves the heat transfer rate, which relates to the heat transfer dynamics and therefore the controller tuning. If the film coefficient is too low, there will be a large thermal lag in the temperature control and therefore the control system will not be very responsive.

If the fluid is flowing in laminar regime, the heat transfer in the direction normal to fluid flow is all by conduction in the fluid.

**Overall Heat Transfer Coefficient**

Let us now consider the common process scheme by which heat is transferred from one fluid to another separated by a metal wall. This is the method of heat transfer done in our lab, experiment 2.

For the model shown, the heat flows through three resistances in series. The overall heat transfer coefficient is defined in terms of the overall temperature difference:

\[ q = UA(T_h - T_c) = h_h A(T_h - T_{wh}) = h_c A(T_{ch} - T_{wc}) = k_m A \frac{T_{wh} - T_{wc}}{\delta} \]

The overall heat transfer coefficient can be calculated from the individual coefficients as a sum of the resistances relationship. This is the same relationship used for electrical resistances in parallel:

\[ \frac{1}{UA} = \frac{1}{h_h A} + \frac{\delta}{k_m A} + \frac{1}{h_c A} \]

\( U \) is the overall coefficient and \( h \) is the individual coefficient contributions. Both have the same units of BTU/hr-ft^2 DegF.
Heat exchangers experience fouling or a build up of scale and corrosion film products that also acts as a thermal resistance to heat flow. These factors are called fouling factors and should also be included in the overall U calculation.

In industrial applications, as well as for our lab heat exchanger, the film coefficient can be calculated. This is a complex relationship involving several dimensionless numbers including Reynolds. This is called the Seider & Tate equation:

$$\frac{hD}{k} = 0.022 \left[ \frac{(DG)}{G} \right]^{0.8} \left[ \frac{C_p \mu}{\mu b} \right]^{0.4} \left[ \frac{\mu_b}{\mu_w} \right]^{0.16}$$

- **h** = The fluid's film heat transfer coefficient
- **D** = Internal pipe diameter
- **k** = Thermal conductivity of the fluid at bulk temperature "b"
- **G** = Mass velocity of the fluid
- **C_p** = Specific heat of the fluid at bulk temperature "b"
- **\mu** = Absolute viscosity of the fluid
- **\mu_b** = The Fluid fluid's viscosity at bulk temperature "b"
- **\mu_w** = The Fluid fluid's viscosity at wall (film) temperature "w"
- **b** = Bulk fluid temperature
Note that this G is different than the G in fluid mechanics, where it is the specific gravity. G is the mass flow rate, pounds per second, and is directly related to the fluid velocity:

$$G = \rho VA$$

Where \(\rho\) is the fluid density. Note how the units cancel:

Pounds/sec = (pounds/ft^3)*(ft/sec)*(ft^2)

*Therefore, we can see that the rate of film heat transfer through a heat exchanger related to the 0.8 power of the fluid velocity. This results in very nonlinear dynamic temperature response.*

A design calculation for our exchanger is available on our web sight and will show that the amount of heat transferred is proportional to the fluid flow rates.

The basic equation of heat transfer is the enthalpy or energy balance. The heat passed from the hot fluid to the cold fluid is also equal to the heat transferred through the exchanger wall.

$$Q = mC_p(\Delta T)$$

$$Q_{ex} = UA(lmdt)$$

$$Q_h = m_hC_p(\Delta T_h)$$

$$Q_c = m_cC_c(\Delta T_c)$$

$$Q_h = Q_c = Q_{ex}$$
\[ Q = \text{Heat Transfer Rate, BTU/hr.} \]
\[ m = \text{mass flow rate, lb./hr.} \]
\[ C_p = \text{Heat Capacity, BTU/lb.} \]
\[ \Delta T = \text{Temperature difference DegF} \]
\[ U = \text{Heat Transfer coefficient, BTU/hr-ft^2 DegF} \]
\[ A = \text{Area ft^2} \]
\[ \text{Imdt} = \text{log mean delta temperature difference for countercurrent flow or just mean temperature difference for co current flow or even passes. This is beyond the scope of the course. Our lab exchanger has an even number of tube passes, 4 and one shell pass, therefore the mean temperature difference calculation is rather complex.} \]

Let's look inside an exchanger and see how a shell and tube exchanger is constructed.

One of the fluids passes through the inside of tubes by entering and exiting the end fitting shown on the front. This is called the Tube Side. The plate shown horizontally in the end fitting is used to direct the flow to half the tubes. This particular exchanger is a two pass the flow enters vertically at the bottom of the end fitting and out the top. The fluid traveling from one end to the other is called a “pass”. The number of trips down the length is called the number of passes.

The horizontal hole in the end fitting is used to insert a thermo well. Note that one tube is larger to accommodate the well. This company implemented this unique design to reduce dead time as well as permit good control at low flows since the temperature is sensed in the exchanger rather than sensed in the outlet pipeline.
This particular illustration is of a steam water heater. The other fluid flows over the outside of the tubes and is called the Shell Side. The vertical internal plates divert the fluid in a ribbon pattern to improve contact.
Our heat exchanger has hot water flowing through the tube side. Passing cold water through the shell side cools this hot water.

This drawing shows the internal temperature distribution in our 4 pass exchanger. Note that the hot water temperature is reduced as it “passes” from one side to the other while the cold side increases across the length. When you run the experiment, you can feel the change in temperature as you run your hand across the outside of the shell.
**Dynamic Response**

Temperature control of a shell and tube heat exchanger is a self-regulating process. Constant flow and temperature through the shell and tube side will result in constant outlet temperatures. Because the shell side has the internal baffles and the tubes "pass" perpendicular across these baffled sections, the outlet temperatures exhibit a distributed parameter behavior. If there is any change in the inlet temperature, the outlet temperature should change too. The following plot shows the results of a step change to shell for our exchanger:

![Exchanger Temperature graph](image)

The tube outlet temperature response for the lab exchanger can be estimated as a first order time constant with dead time. The time constant can be calculated for this exchanger as:

$$\tau = \frac{MC_p}{UA}$$

*M is the mass of tube and shell contents
*Cp is the specific heat
*U is the overall heat transfer coefficient
*A is the tube area*
ChE 433 Process Control Laboratory Experiment 2

For our experiments we are going to control the temperature of the hot water discharge. Our hot water comes from the University at about 60 DegC. We are experimenting with ways to cool this water to ~ 35 DegC by using cold water, that currently is operating at 20 to 25 DegC. I suspect that the entire experiment will exhibit different dynamics during the winter when the cold-water temperature will drop considerably. This seasonal variation is experienced in many industrial process control applications.