Measurement of Axial Thrust Number
in Various Agitation Flow Regimes

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Abstract

Structural design of agitators requires a detailed knowledge of the relationship between mixing strength and the axial force generated by the fluid. A bench-scale baffled agitation vessel with a dish bottom and standard tank dimensions was instrumented with an electronic load cell and optical tachometer to characterize the dimensionless impeller Reynolds number and axial thrust number across Reynolds numbers from $1\times10^6$ using marine propeller, HE–3, and P–4 impellers.
1 Introduction

Agitation is broadly used in chemical engineering processes to ensure proper mixing [1]. Mixing is important for both mass and heat transfer operations, such as chemical reactors [2].

The mechanical design of agitation equipment is generally based on heuristics from experience rather than experimental data. This leads to overdesign, increased production cost, and decreased efficiency [1]. In particular, the relationship between operational parameters and the axial force exerted by the fluid on the motor mount is virtually unknown. As a result, large, costly structures are used to support the agitation unit to prevent mechanical failure during mixing.

The degree of mixing in an agitation vessel is characterized by an impeller Reynolds number ($N_{Re}$). This parameter is used to estimate blending time, power demand, and vessel sizing for agitation vessel scale-up [1].

We have therefore devised a novel method for measuring the axial thrust generated during mixing across five decades of $N_{Re}$. These data will aid in the development of improved structural design of agitation equipment.

2 Methods

2.1 Apparatus

A baffled, flat-bottom acrylic tank (model 4F63; Chemineer, Dayton, OH) was placed on an electronic load cell (HW-G; A&D Mercury Pty Ltd., Thebarton, Australia). The load cell had a resolution of 0.1 N. The tank was filled with corn syrup (Karo, Louisville, KY). The syrup was agitated at a variety of angular velocities between 1–100 Hz using a motor with variable gear ratio (model BP11; Chemineer) using H-3, P-4, and marine impellers (Chemineer). The angular velocity was measured using an optical tachometer (EW-87799096, Cole-Parmer)
Instrument Company, Vernon Hills, IL). Standard agitation tank dimensions were used (see McCabe et al., 2001, p. 463).

2.2 Calibration

The load cell was calibrated by incrementally increasing the volume of water in a five-gallon bucket on the cell. The output was recorded at each increment, allowing a determination of the cell’s sensitivity (output counts per unit change in mass). This process was repeated three times. The density of the water was measured after each iteration to account for potential temperature effects.

The optical tachometer’s output was verified by visually counting the rotation rate of the agitator shaft for one minute and comparing this count to the value reported on the tachometer.

2.3 Materials

Karo® Corn Syrup (ACH Food Companies, Inc., Cordova, TN) was used without further purification. Serial dilutions were performed to achieve different viscosities with 0, 0.01, 0.1, and 1 parts deionized water per volume part corn syrup.

2.4 Viscosity Measurement

The kinematic viscosity $\nu$ of the solutions were characterized using creeping flow capillary rheometry modeled by the Haagen-Pouiseuille equation,

$$\nu = \frac{\pi g R^4}{8 L V t},$$

(1)

where $g$ is the acceleration due to gravity, $R$ is the radius of the capillary, $L$ is the length of the flow path, $V$ is the volume of the tube, and $t$ is the time required for the fluid to flow across the measurement length $L$. Note that the fractional term $\pi g R^4/8 L V$ is a constant.
available in calibration tables. Then, the Newtonian viscosity $\mu$ may be computed as

$$\mu = \nu \rho,$$  \hspace{1cm} (2)

where $\rho$ is the fluid density.

3M Neutral Cleaner (3M Corp., St. Paul, MN) was used to remove any remnants of fluid used in previous tests. The tubes were then rinsed repeatedly with water and finally with acetone, then blown dry with air. Cannon-Fenske 100/406A and 100/G139 kinematic viscometers (Cannon Instrument Company, State College, PA) were used. The end of the fill tube was submerged in the fluid to be used and suction was then applied to the opposing tube until the fluid reached the fill line. The tube was then removed from the fluid, inverted, and placed in a level holder. Suction was then applied to the fill tube until the fluid rose above the draw line. The fluid level was then allowed to decrease to the draw line. The time required for the fluid meniscus to travel from the draw line to the fill line is the efflux time $t$.

### 2.5 Nondimensional Parametrization

Thrust $F_t$ and angular velocity $n$ were nondimensionalized respectively as the axial thrust number $N_T$,

$$N_T = \frac{F_t}{\rho n^2 D_a^4},$$  \hspace{1cm} (3)

and $N_{Re}$,

$$N_{Re} = \frac{D_a^2 n \rho}{\mu},$$  \hspace{1cm} (4)

where $D_a$ is the impeller diameter.
2.6 Statistical Methods

An analysis of variance was performed to determine whether $N_t$ varied between impeller types. Linear regression was used to determine dependence of $N_t$ with $N_{Re}$ in each flow regime. Significance was assumed at the level $\alpha < 0.05$.

3 Results

The load cell’s output depended linearly on the mass of water (Fig. 1). The tachometer’s measurement agreed with manual counting for all observations (Fig. 2), so this trend was extrapolated to higher angular velocities. The viscosity of the sample varied as a power law as the corn syrup was diluted (Fig. 3).

The $N_t$ values for the H–3 impeller and marine propeller were very similar at all values of $N_{Re}$, while $N_t$ for the P–4 impeller was generally higher (Figs. 4–6). This difference achieved statistical significance ($p << 0.001$) in the fully turbulent plateau. Linear correlation coefficients for each flow regime are given in Table 1.

4 Discussion

For all impeller types tested, $N_t$ exhibited three trends:

1. linear decrease with increasing $N_{Re}$ in the laminar flow regime ($N_{Re} < 50$);
2. linear increase with increasing $N_{Re}$ in the transitional regime ($50 < N_{Re} < 1000$); and
3. remained constant at all $N_{Re}$ in the fully turbulent regime ($N_{Re} > 1000$).

The fully turbulent results for the P–4 and H–3 impeller matched well with the data of [3] and Scaba data [1], respectively. The Scaba impeller is similar to the HE–3 used in the present study and is therefore used as a basis for comparison.
The P–4 impeller type is designed to maximize axial flow in fully turbulent mixing. This agrees with the observation that it has the highest turbulent $N_t$.

Significant errors were present at low angular velocities. These errors are the result of uncertainties in the measurement of $F_t$: these forces necessarily oscillated due to the rotation of the impeller. Future studies might employ a moving averaging technique to smooth the $F_t$ data, thereby decreasing this uncertainty.

The correlation between dimensionless quantities presented here allow direct calculation of the axial force due to agitation, $F_t = N_t \rho n^2 D_a^4$, at a given $N_{Re}$ for the given tank dimensions. Note that using these correlations for unbaffled or dish-bottom tanks, in systems with non-Newtonian fluids, or vessels with non-standard geometries is likely to give erroneous results. Future studies will attempt to determine the dependence of these correlations on these other aspects of the agitation process and vessel design.

References


Appendix

Sample Calculations

Note: sample calculations may also be submitted in hand-written form to the TA.

Calculation of $N_t$ from Eqn. 3 These calculations are based on measurements of $F_t = 0.16 \text{ lb}_f$, $\rho = 8.345 \text{ lb}_m/\text{gal}$, $n = 12.3 \text{ RPM} = 10.5 \text{ rad/s}$, and $D_a = 4.00 \text{ in} = 0.102 \text{ m}$. Converting each of these to SI units yields $F_t = 0.16 \text{ lb}_f = 0.71 \text{ N}$, $\rho = 1000 \text{ kg/m}^3$, $n = 1.29 \text{ rad/s}$, and $D_a = 0.102 \text{ m}$. The axial thrust number calculation is then as follows:

$$N_T = \frac{F_t}{\rho n^2 D_a^4} = \frac{0.71}{1000 \times 1.29^2 \times 0.102^4} = \frac{0.71}{0.1801} = 3.94$$

(5)
Calculation of $N_{Re}$ from Eqn. 4  These calculations are based on measurements of $D_a$, $\rho$, and $n$ as stated in the previous section. Viscosity $\mu$ was calculated from experimental observations with a mean efflux time $t_e = 17.3$ s in a viscometer with a calibration constant $c$ of 0.8 cSt $-s^{-1}$. Converting $c$ to SI units yields $c = 8 \times 10^{-7}$ m$^2$/s. This gives a kinematic viscosity $\nu$ according to the rheometer equation

$$\nu = ct = 8 \times 10^{-7} \times 17.3 = 1.38 \times 10^{-5} \text{ m}^2/\text{s}. \quad (6)$$

The viscosity $\mu$ may then be calculated as

$$\mu = \nu \rho = 1.38 \times 10^{-5} \times 1000 = 1.38 \times 10^{-2} \text{ Pa} \cdot \text{s}. \quad (7)$$

With these values in hand, $N_{Re}$ is calculated as

$$N_{Re} = \frac{D_a^2 n \rho}{\mu} = \frac{0.102^2 \times 1.29 \times 1000}{1.38 \times 10^{-2}} = \frac{13.4}{1.38 \times 10^{-2}} = 973. \quad (8)$$
Table 1: Correlation Coefficients for Linear Regressions of the Form $N_t = mN_{Re} + b$

<table>
<thead>
<tr>
<th></th>
<th>Marine Propeller</th>
<th>HE-3</th>
<th>P-4</th>
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<tbody>
<tr>
<td>$Re &lt; 50$</td>
<td>0.8447</td>
<td>-0.0233</td>
<td>0.9004</td>
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<tr>
<td>50 $&lt; Re &lt; 1000$</td>
<td>0.0648</td>
<td>0.0007</td>
<td>0.1096</td>
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<tr>
<td>$Re &gt; 1000$</td>
<td>0.3272</td>
<td>0.0000</td>
<td>0.3091</td>
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Figure 1: Calibration data for the load cell. Data points indicate the mean load cell output for each increment in water mass, while error bars indicate the standard deviation. The slope of the best-fit regression line corresponds to the sensitivity of the load cell.

$y = 5073.x + 885.2$

$R^2 = 0.995$
Figure 2: Calibration data for the optical tachometer. Linear regression indicated a slope of unity with zero offset, indicating exact agreement between the true angular velocity and the tachometer output.
$y = 4.802x^{36.89}$
$R^2 = 0.997$

Figure 3: Viscosity as a function of water content.
Figure 4: Axial thrust number as a function of Reynolds number for the marine propeller.
Figure 5: Axial thrust number as a function of Reynolds number for the HE–3 impeller.
Figure 6: Axial thrust number as a function of Reynolds number for the P–4 impeller.