Let $V_1 = (1, 3, -1)$
$V_2 = (2, 5, -9)$
be two vectors in $\mathbb{R}^3$.

1. Calculate $V_1 \times V_2$

2. Calculate $\theta$ the angle between $V_1$ and $V_2$ by using the formula
   \[
   \cos \theta = \frac{V_1 \cdot V_2}{||V_1|| \, ||V_2||}
   \]
   Hence calculate $\sin \theta$.

3. Calculate $||V_1 \times V_2||$ and compare your result with 2.

Let $V_3 = (-3, 2, 7)$

4. Calculate
   \[
   V_1 \times (V_2 \times V_3) \quad \text{and} \quad (V_1 \times V_2) \times V_3
   \]
   Are they equal??
5. Calculate
\[ |(V_1 \times V_2) \cdot V_3| \]
the magnitude of the scalar triple product of \(V_1, V_2, V_3\). This number is the volume of a parallelepiped.

6. Show that
\[ V_1 \times (V_2 \times V_3) = (V_1 \cdot V_3) V_2 - (V_1 \cdot V_2) V_3 \]
and
\[ (V_1 \times V_2) \times V_3 = (V_1 \cdot V_3) V_2 - (V_2 \cdot V_3) V_1 \]
This is the vector triple product.

7. Show that
\[ V_1 \times (V_2 \times V_3) + V_2 \times (V_3 \times V_1) + V_3 \times (V_1 \times V_2) = 0 \]

8. Show that
\[ \frac{11 a_1}{\sin A} = \frac{11 b_1}{\sin B} = \frac{11 c_1}{\sin C} \]
Hint: Calculate $\mathbf{c} \times \mathbf{a}$ and $\mathbf{c} \times \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$ and use your judgement.

Challenge problem (I have never done this before)

Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be four vectors in $\mathbb{R}^3$.
Prove the Lagrange identity

$$(\mathbf{v}_1 \times \mathbf{v}_2) \cdot (\mathbf{v}_3 \times \mathbf{v}_4) = (\mathbf{v}_1 \cdot \mathbf{v}_3)(\mathbf{v}_2 \cdot \mathbf{v}_4) - (\mathbf{v}_1 \cdot \mathbf{v}_4)(\mathbf{v}_2 \cdot \mathbf{v}_3)$$

Less challenging problem

Show that volume of this tetrahedron is

$$V = \frac{1}{6}[\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})]$$

Hint: $V$ is one third of the product of the area of its base and its vertical height.