



# Lecture 2

## CMOS Circuits

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- Threshold Voltage

$$V_T = V_{T0} + \gamma \left( \sqrt{|-2\phi_F| + V_{SB}} - \sqrt{|-2\phi_F|} \right)$$

$$\phi_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

- I-V Curve

- linear/triode 
$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{gs} - V_{th}) V_{ds} - \frac{V_{ds}^2}{2}$$

- saturation 
$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

- Parasitic Capacitance

- gate cap

- junction cap

$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2} \left( \frac{N_A N_D}{N_A + N_D} \right)} \frac{1}{\sqrt{\phi_0}}$$

# CMOS Capacitance



- Overlap (gate) Cap
  - $C_{gso}$
- Junction Cap
  - $C_{diff}$

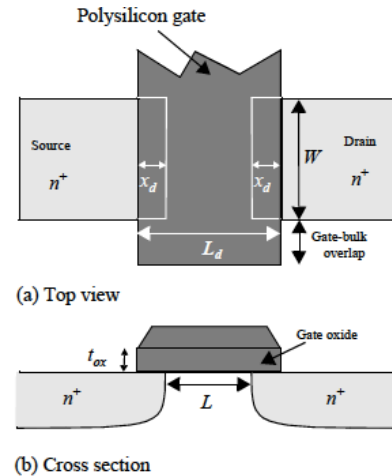


Figure 3.28 MOSFET overlap capacitance.

$$C_{diff} = C_{bottom} + C_{sw} = C_j \times AREA + C_{jsw} \times PERIMETER$$

$$= C_j L_S W + C_{jsw} (2L_S + W)$$

$$C_{GSO} = C_{GDO} = C_{ox} x_d W = C_o W \quad (3.45)$$

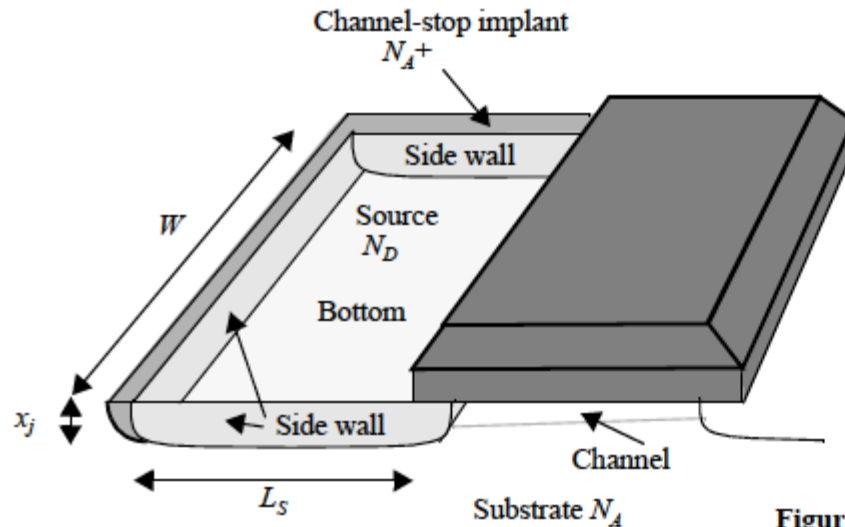


Figure 3.32 Detailed view of source junction.



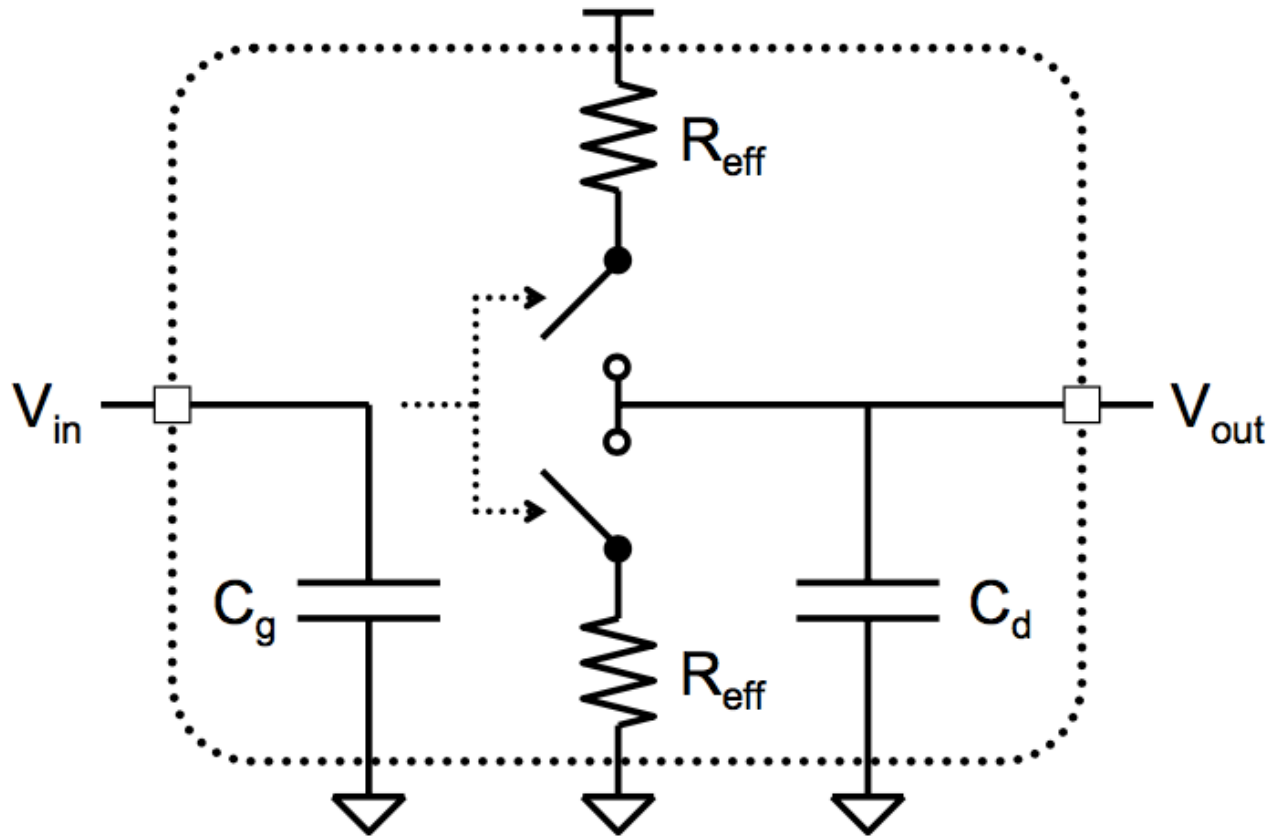
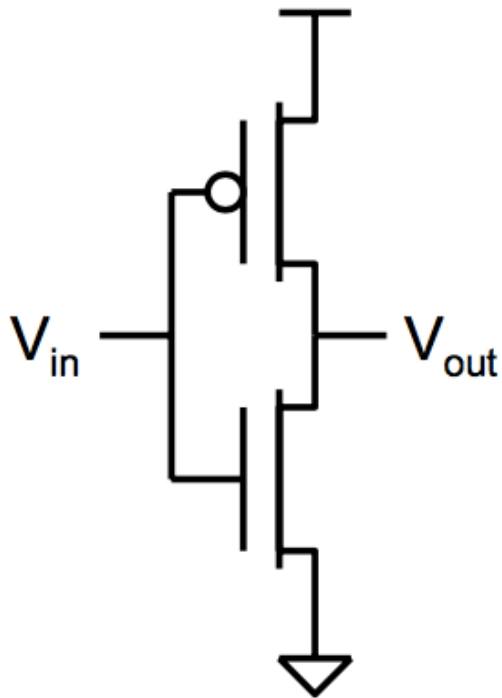
CMOS Inverter

Combinational Logic

Sequential Circuits

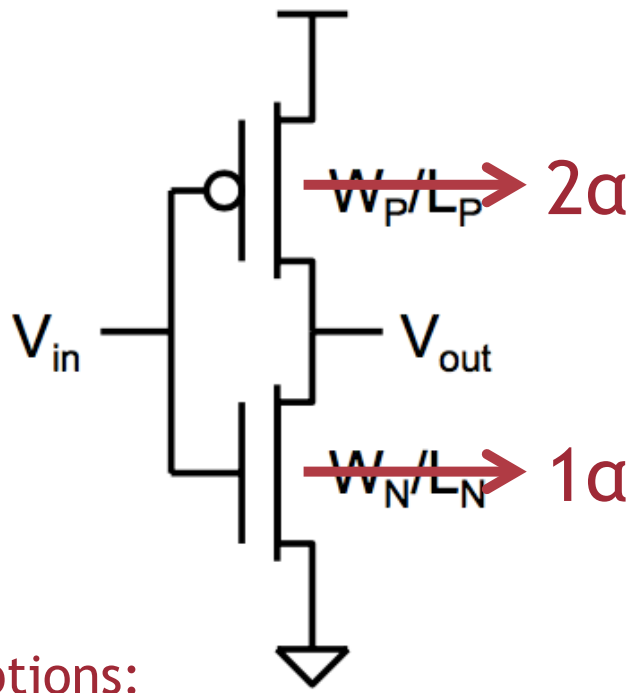
Memory

# Inverter: Simple RC Model



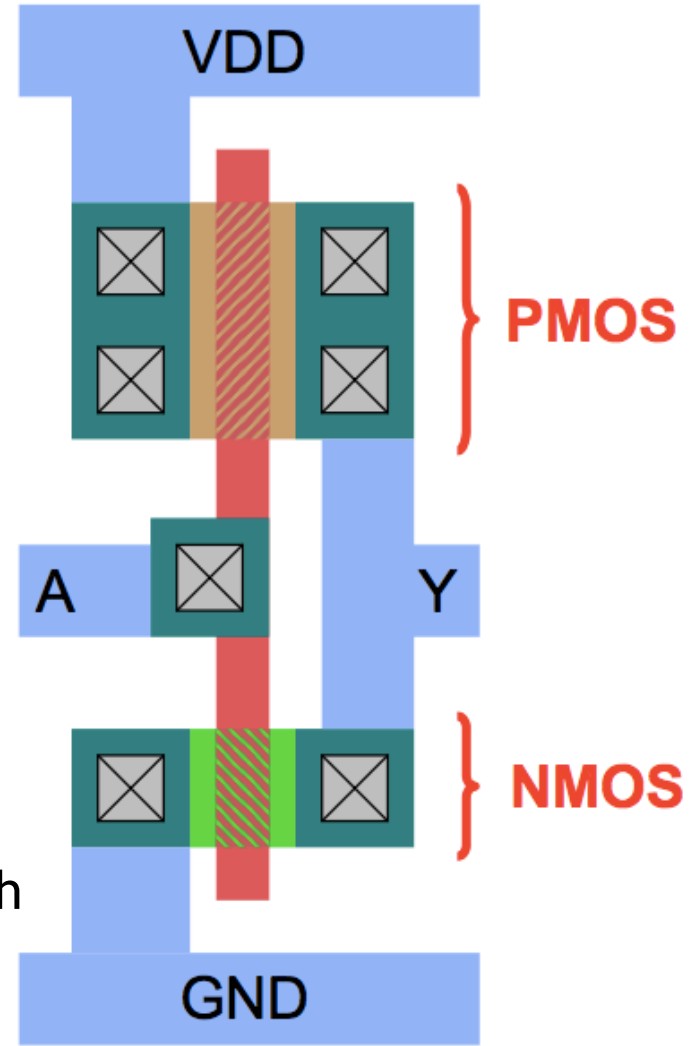
$$R_{\text{eff}} = R_{\text{eff,N}} = R_{\text{eff,P}}$$
$$C_g = C_{g,N} + C_{g,P}$$
$$C_d = C_{d,N} + C_{d,P}$$

# Inverter Layout



## Assumptions:

1. All transistors are minimum length
2. All gate have equal rise/fall time (PMOS twice as wide)
3. Normalize transistor width to minimum NMOS



# Inverter Voltage Transfer Characteristics (VTC)

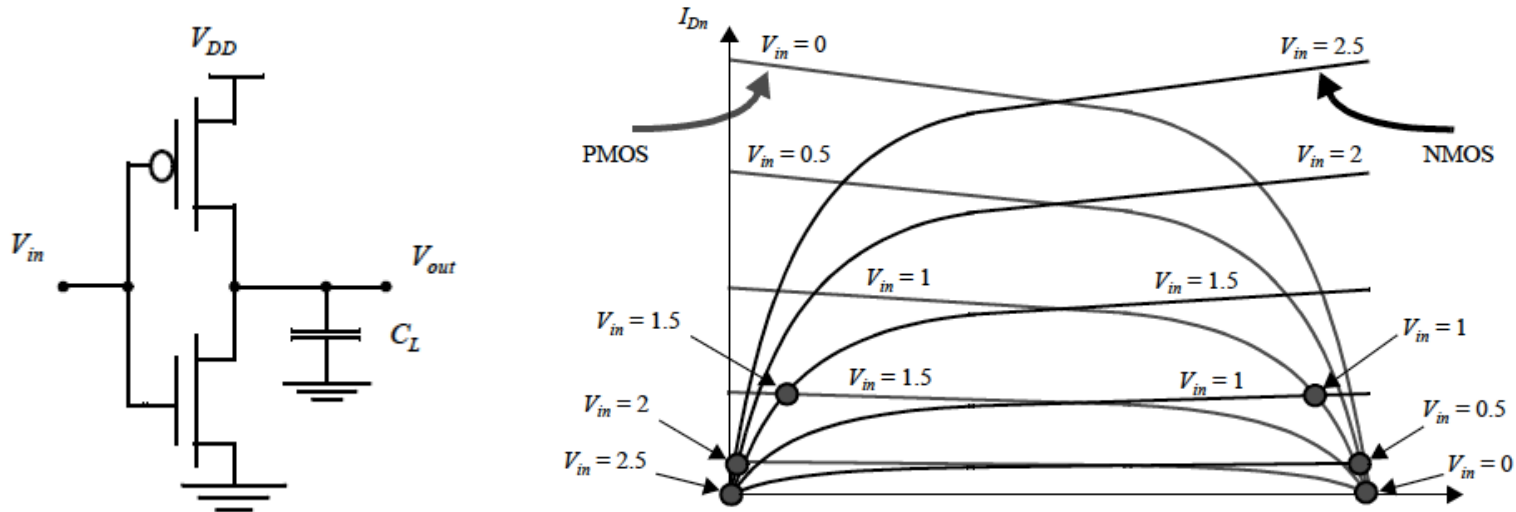


Figure 5.4 Load curves for NMOS and PMOS transistors of the static CMOS inverter ( $V_{DD} = 2.5$  V). The dots represent the dc operation points for various input voltages.

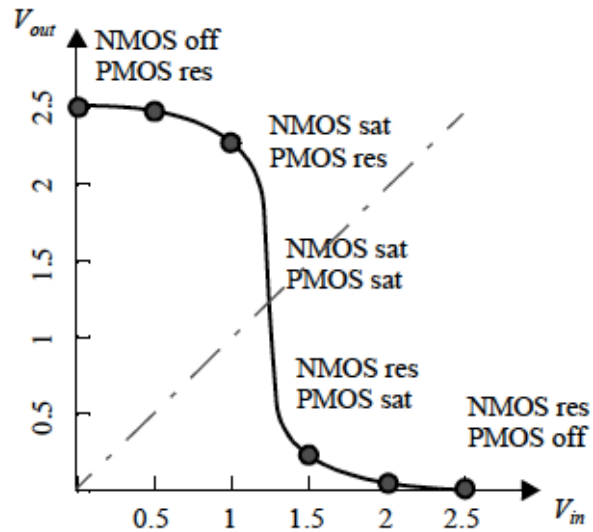
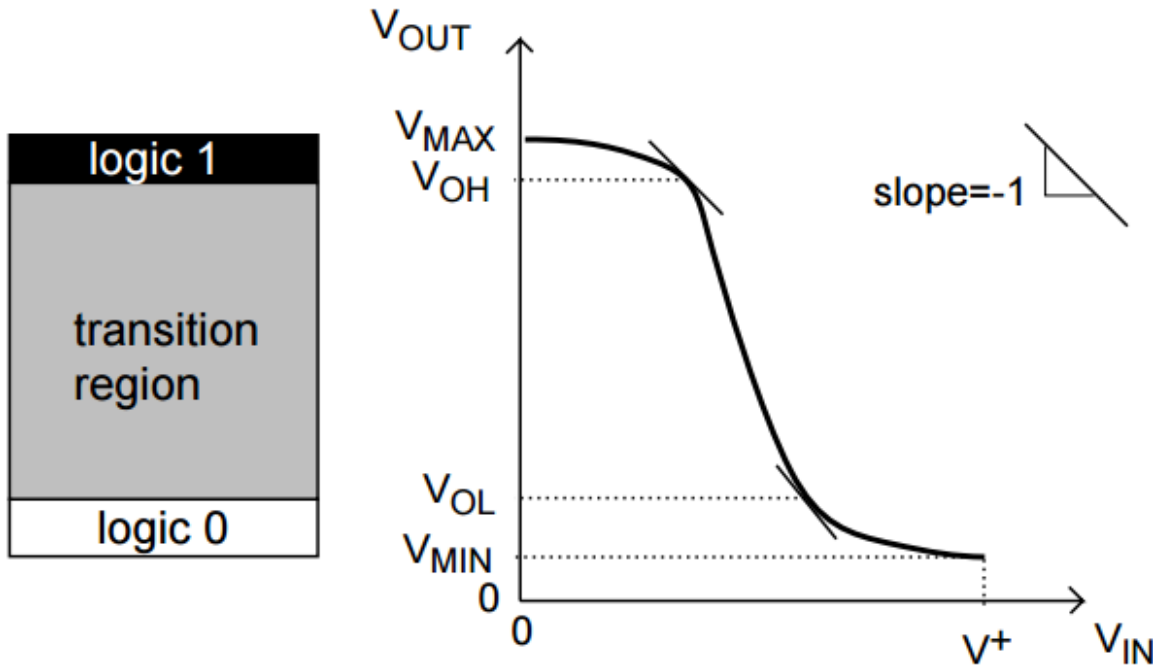


Figure 5.5 VTC of static CMOS inverter, derived from Figure 5.4 ( $V_{DD} = 2.5$  V). For each operation region, the modes of the transistors are annotated — off, res(istive), or sat(urated).

# Inverter Noise Margin



- $V_{IH}$ ,  $V_{IL}$ ,  $V_{OH}$ ,  $V_{OL}$  Definition
  - where the  $dV_{out}/dV_{in}$  slope is -1



- Noise Margin
  - if range of output  $V_{OL}$  to  $V_{OH}$  is wider than the range of input values  $V_{IL}$  to  $V_{IH}$ , then the inverter exhibits noise immunity. ( $|Voltage\ Gain| > 1$ )



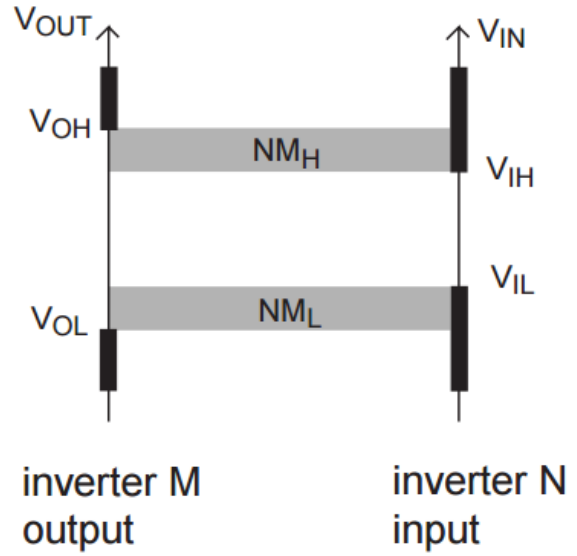
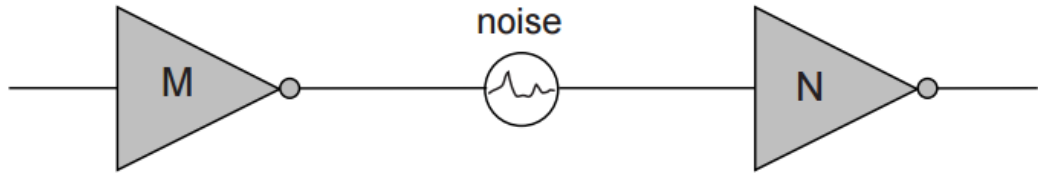
# Inverter Noise Margin



- NMH, NML Definition

$$NM_H \equiv V_{OH} - V_{IH}$$

$$NM_L \equiv V_{IL} - V_{OL}$$



- Capacitive Power

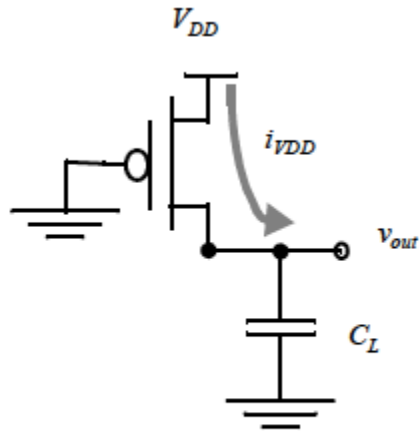


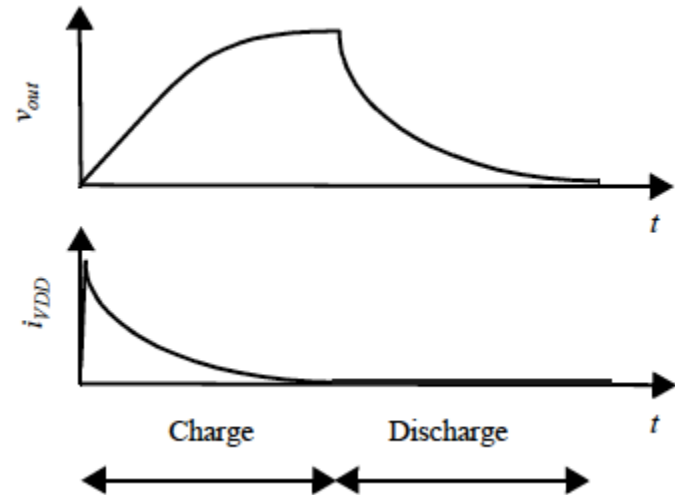
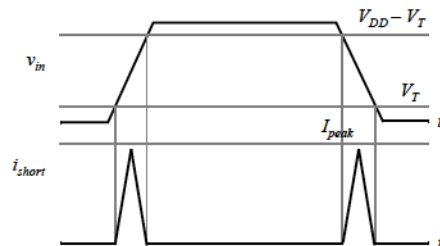
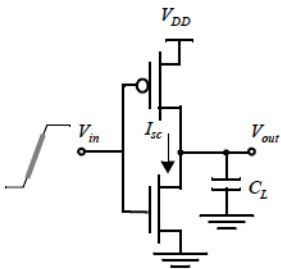
Figure 5.25 Equivalent circuit during the low-to-high transition.

$$E_{VDD} = \int_0^{\infty} i_{VDD}(t) V_{DD} dt = V_{DD} \int_0^{\infty} C_L \frac{dv_{out}}{dt} dt = C_L V_{DD} \int_0^{V_{DD}} dv_{out} = C_L V_{DD}^2$$

$$E_C = \int_0^{\infty} i_{VDD}(t) v_{out} dt = \int_0^{\infty} C_L \frac{dv_{out}}{dt} v_{out} dt = C_L \int_0^{V_{DD}} v_{out} dv_{out} = \frac{C_L V_{DD}^2}{2}$$

$$P_{dyn} = C_L V_{DD}^2 f_{0 \rightarrow 1} = C_L V_{DD}^2 P_{0 \rightarrow 1} f = C_{EFF} V_{DD}^2 f$$

- Short-circuit Power



# Inverter Static Power



- Sub-threshold Leakage

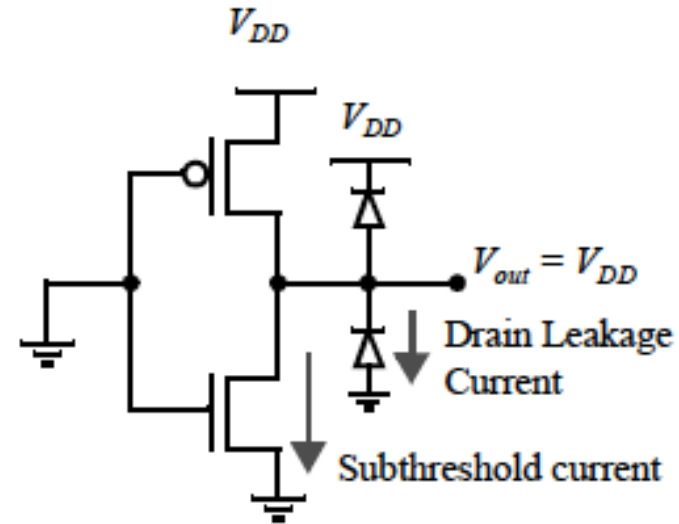
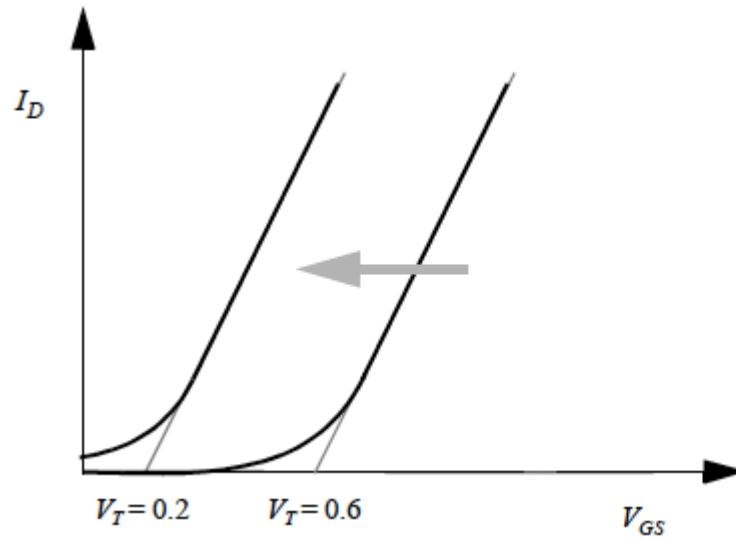
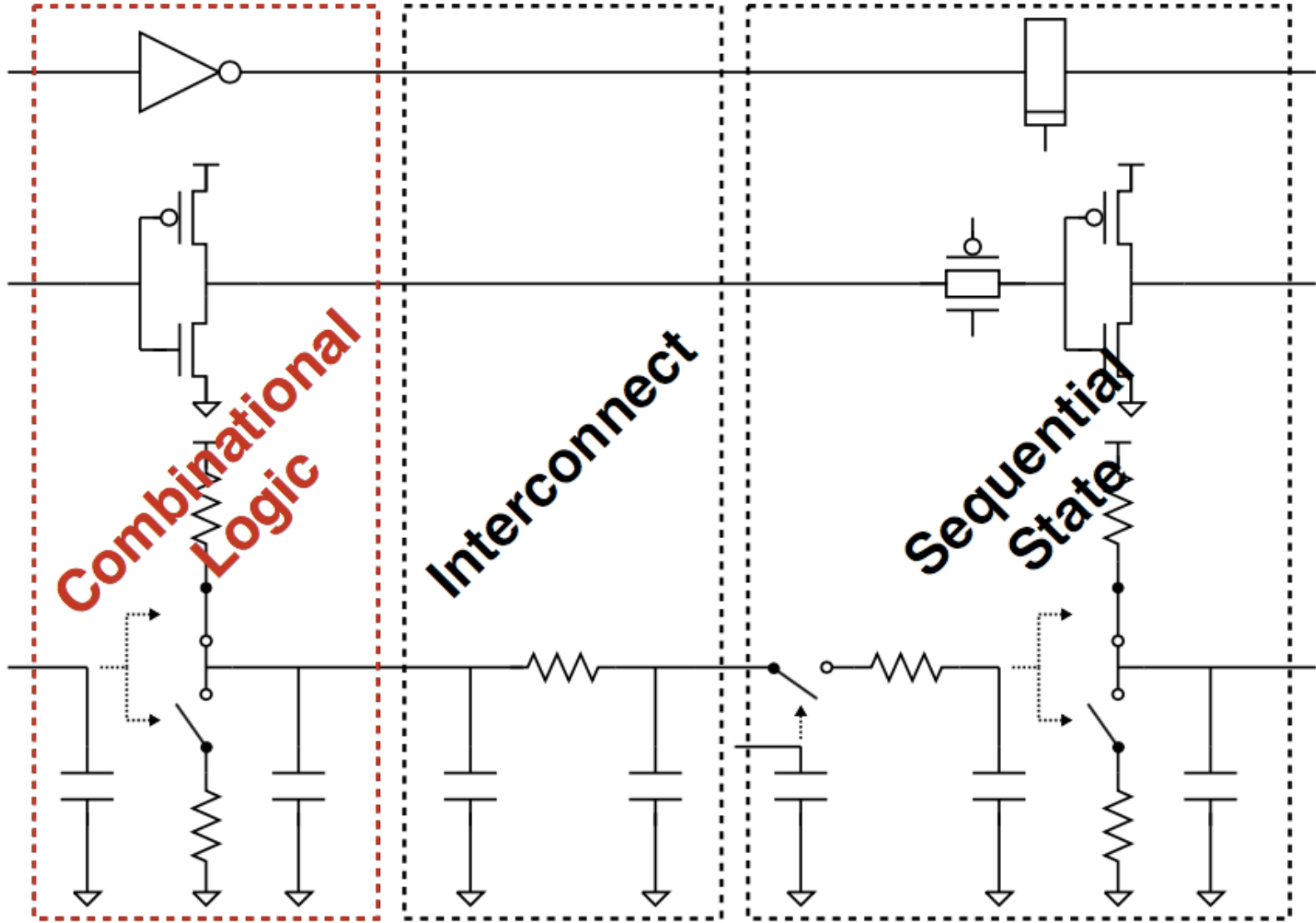
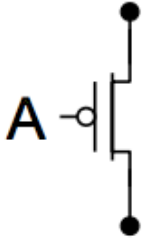


Figure 5.35 Decreasing the threshold increases the subthreshold current at  $V_{GS} = 0$ .

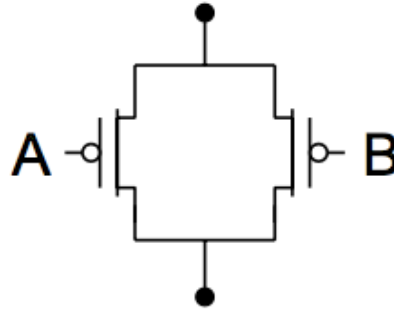
# Outline



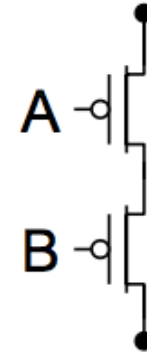
# CMOS Duality



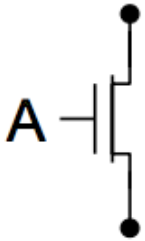
Conducts if  $A=0$



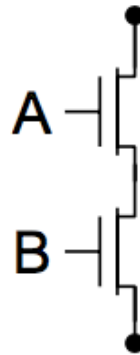
Conducts if  $A=0$  **OR**  $B=0$



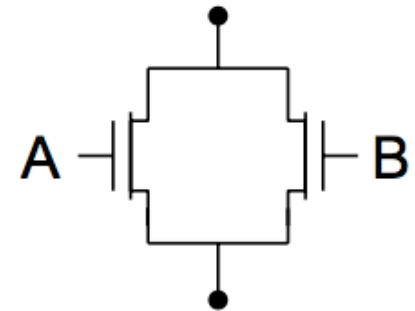
Conducts if  $A=0$  **AND**  $B=0$



Conducts if  $A=1$



Conducts if  $A=1$  **AND**  $B=1$

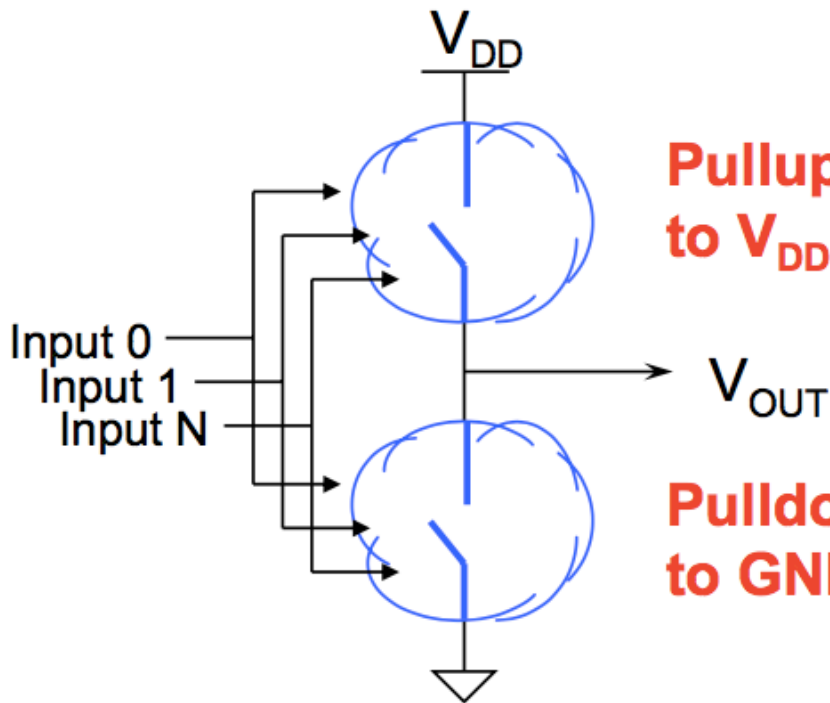


Conducts if  $A=1$  **OR**  $B=1$

# CMOS Static Logic Style



- For every set of input logic values, either pull-up or pull-down network connects to  $V_{DD}$  or GND
  - power rails would be shorted if both connected
  - output would float (tri-state logic), if neither connected



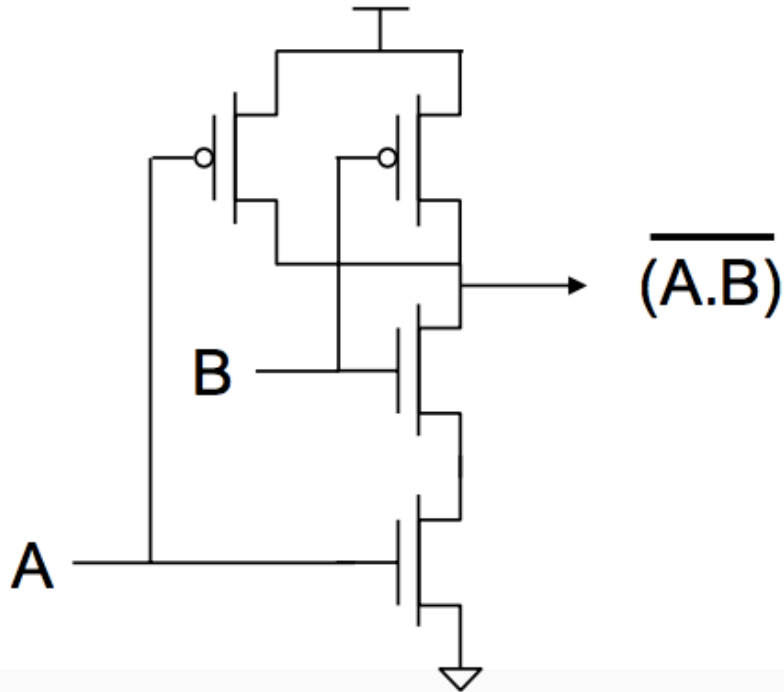
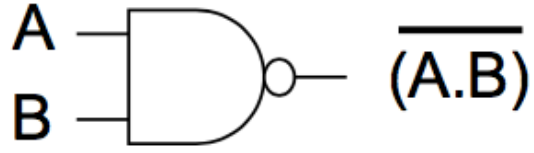
**Pullup network, connects output to  $V_{DD}$ , contains only PMOS**

**Pulldown network, connects output to GND, contains only NMOS**

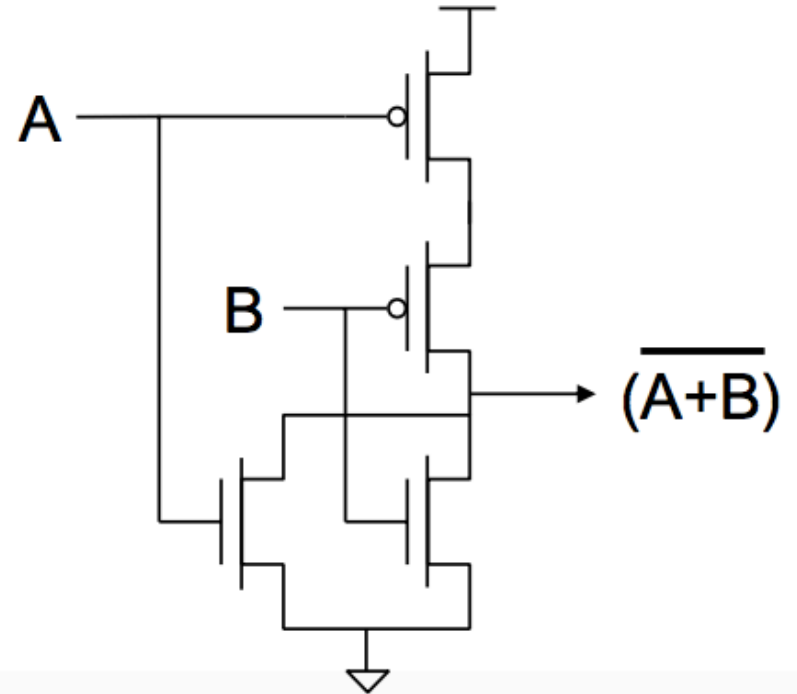
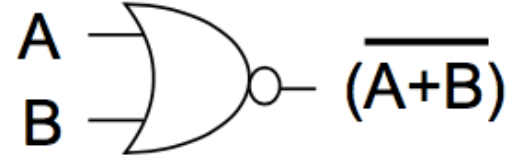
# NAND/NOR Static Logic Gates



## NAND Gate



## NOR Gate



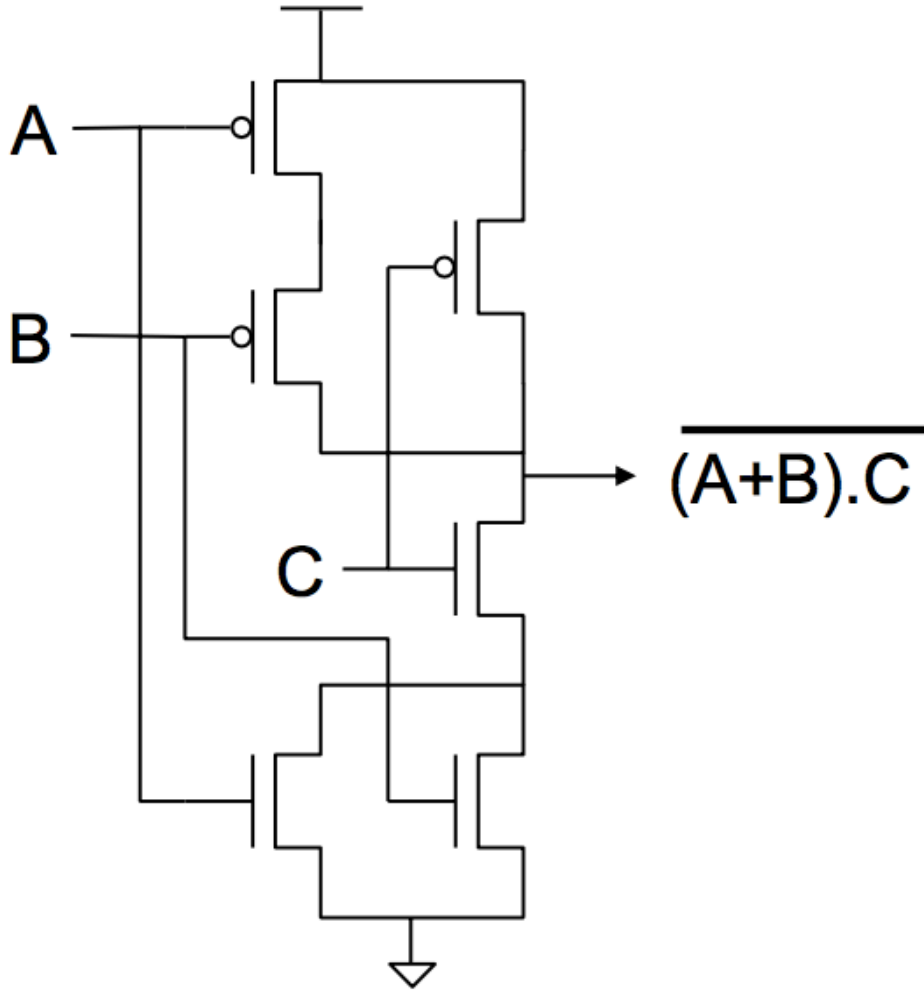
# Design More Complex Gates



- Goal is to create a logic function  $f(x_1, x_2, \dots)$ 
  - can only implement inverting logic with one stage
- Implement pull-down network
  - write  $PD = /f(x_1, x_2, \dots)$
  - use parallel NMOS for OR of inputs
  - use series NMOS for AND of inputs
- Implement pull-up network
  - write  $PU = f(x_1, x_2, \dots) = g(/x_1, /x_2, \dots)$
  - use parallel PMOS for OR of inverted inputs
  - use series PMOS for AND of inverted inputs



# Complex Logic Gate Example

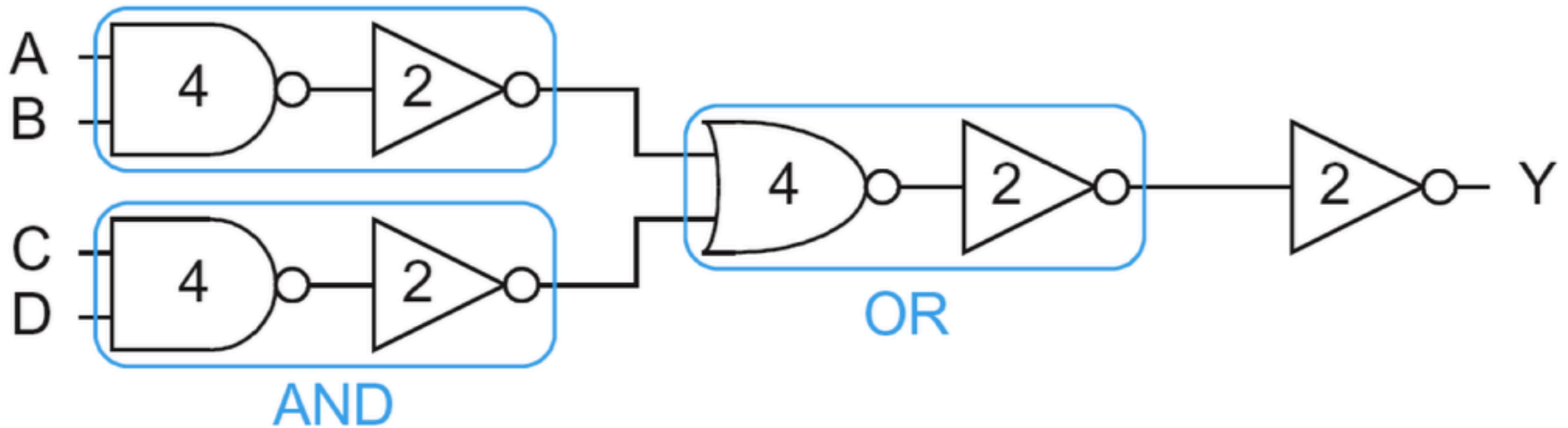
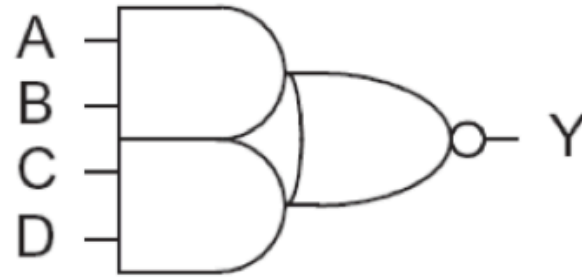
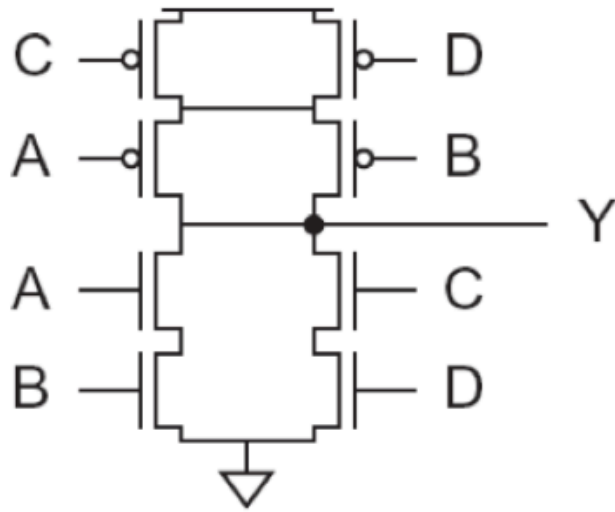


$$f = \overline{(A + B) \cdot C}$$

$$PD = (A + B) \cdot C$$

$$\begin{aligned} PU &= \overline{(A + B) \cdot C} \\ &= \overline{(A + B)} + \overline{C} \\ &= (\overline{A} \cdot \overline{B}) + \overline{C} \end{aligned}$$

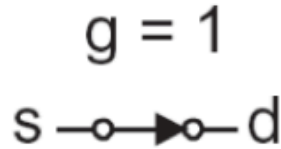
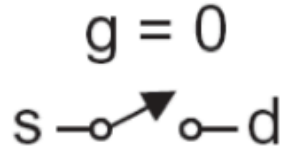
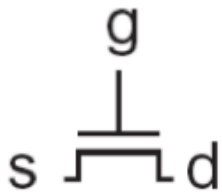
# Multi-Stage Static Logic



# CMOS Pass-Gate Logic Style



nMOS

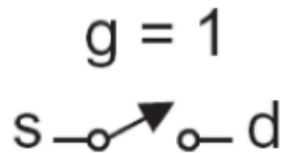
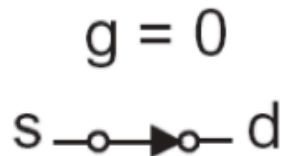
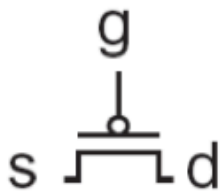


Input  $g = 1$  Output

0  $\rightarrow$  strong 0

1  $\rightarrow$  degraded 1

pMOS

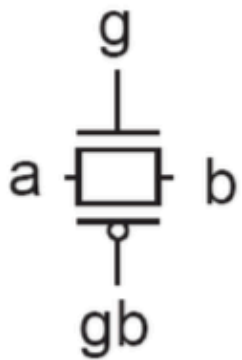


Input  $g = 0$  Output

0  $\rightarrow$  degraded 0

1  $\rightarrow$  strong 1

# CMOS Transmission Gate Multiplexer



$g = 0, gb = 1$



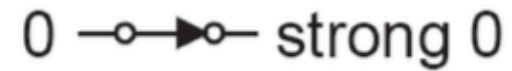
$g = 1, gb = 0$



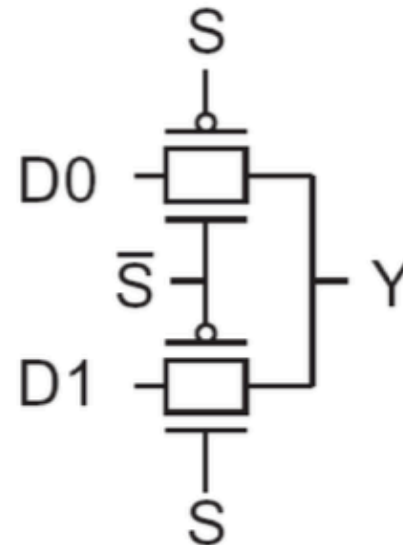
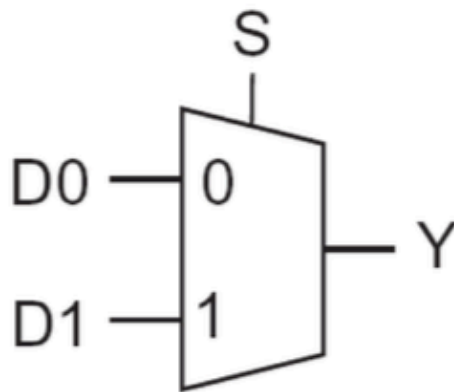
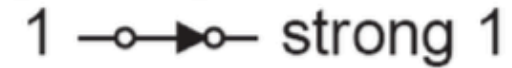
Input

Output

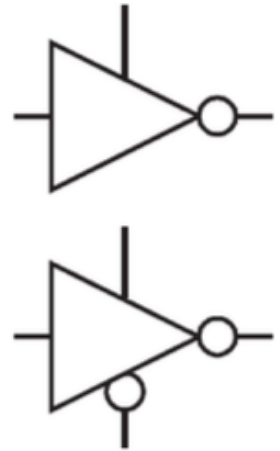
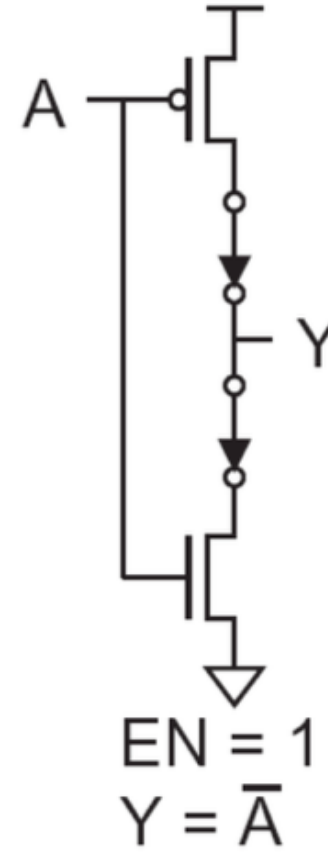
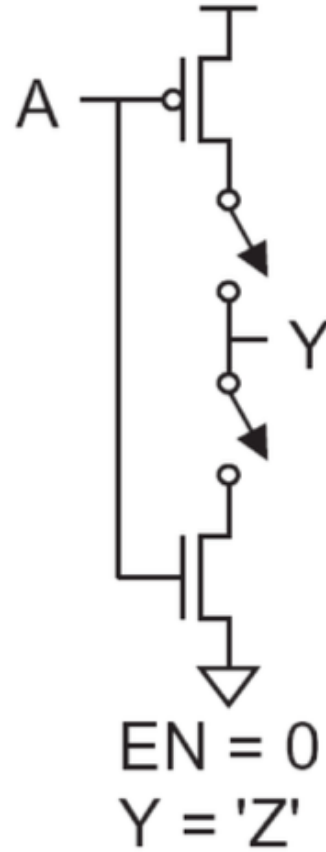
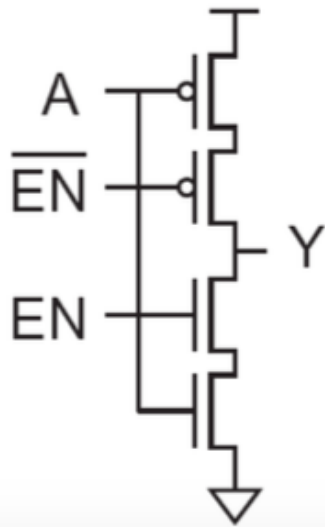
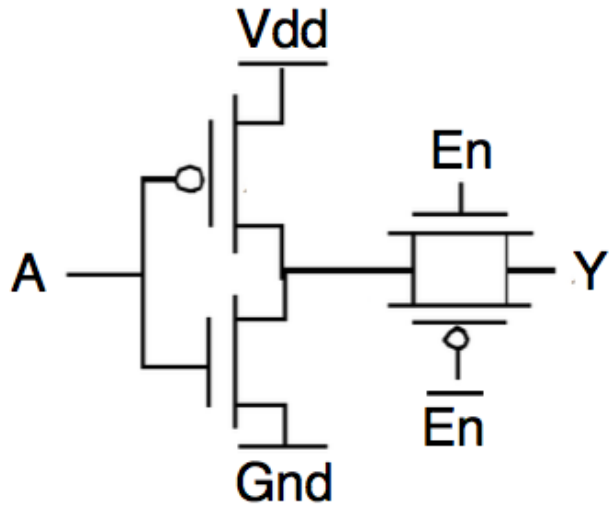
$g = 1, gb = 0$



$g = 1, gb = 0$



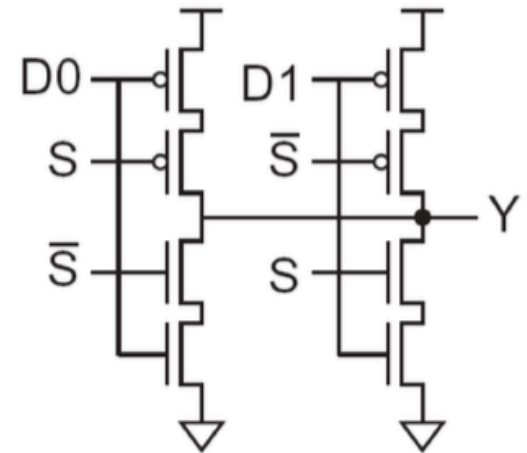
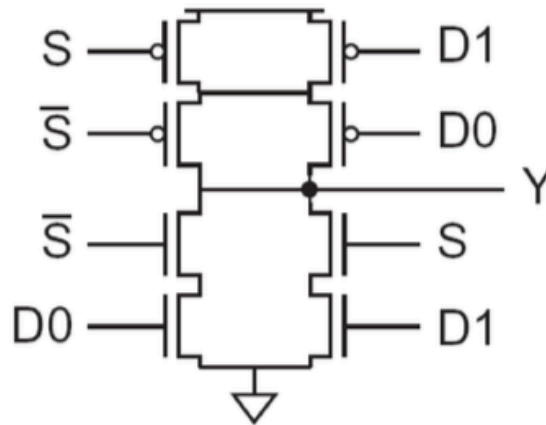
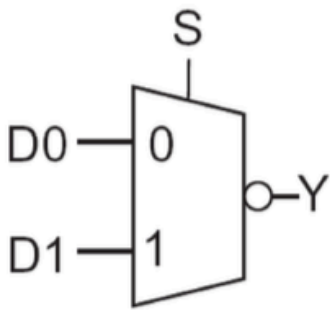
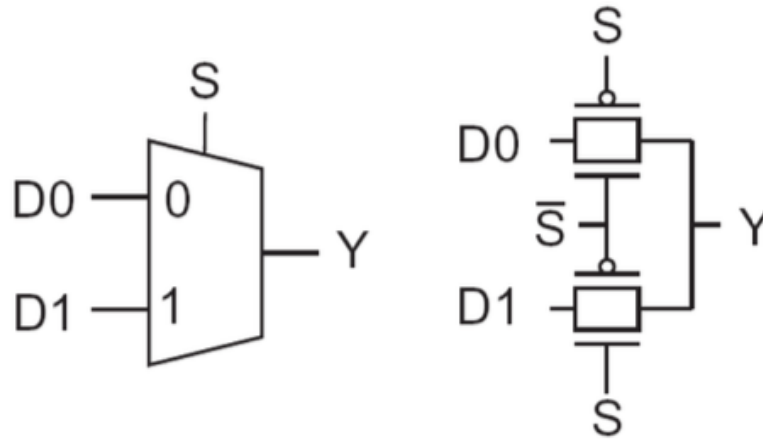
# CMOS Tri-State Buffers



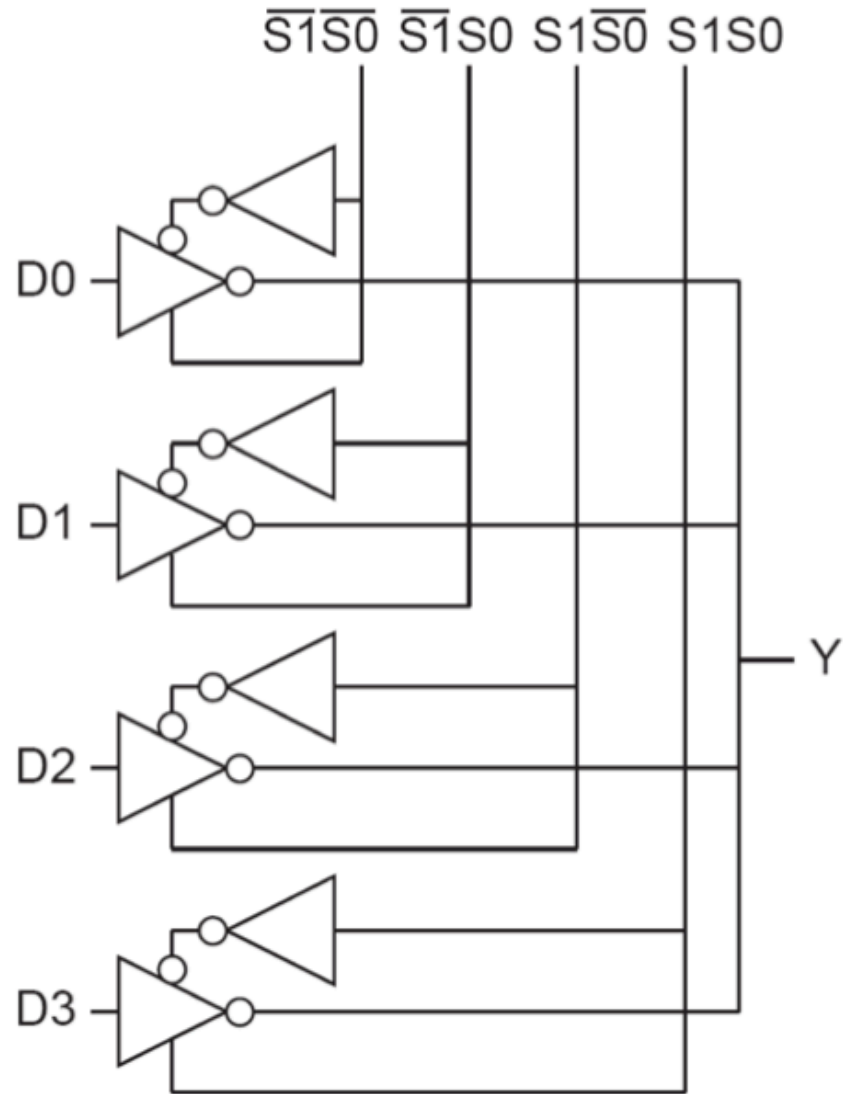
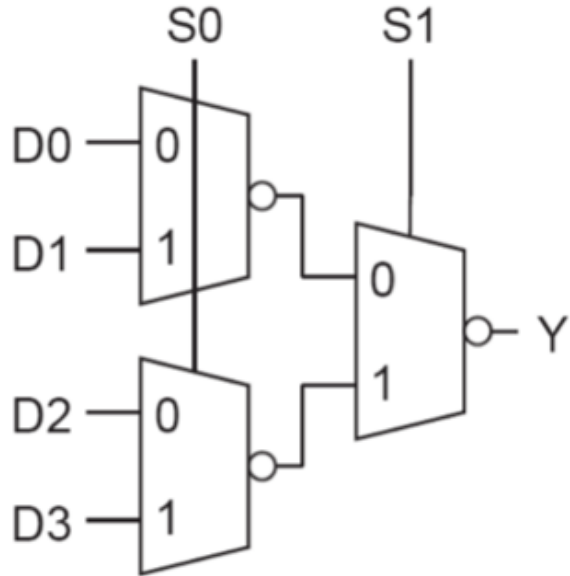
# Various Multiplexer Implementations



- Delay, Area, Energy Trade-offs
  - simple first-order analysis



# Larger Tri-State Multiplexers



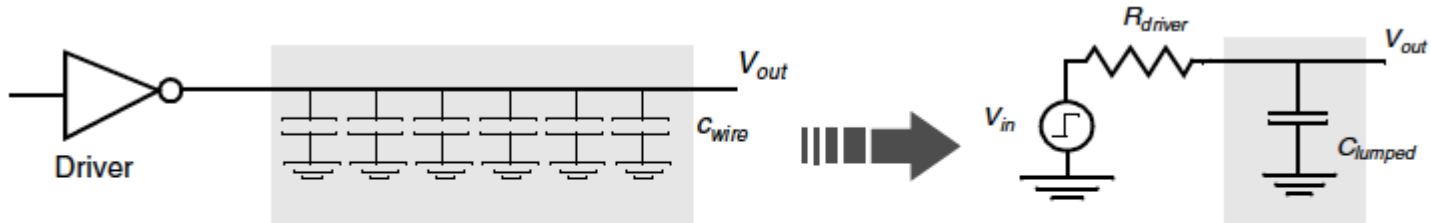
# Homework #2



- Posted on class website
- Due on 1/30 at 2:30pm
- Provide extension to the lecture



- Lumped Model
  - C only
  - RC model

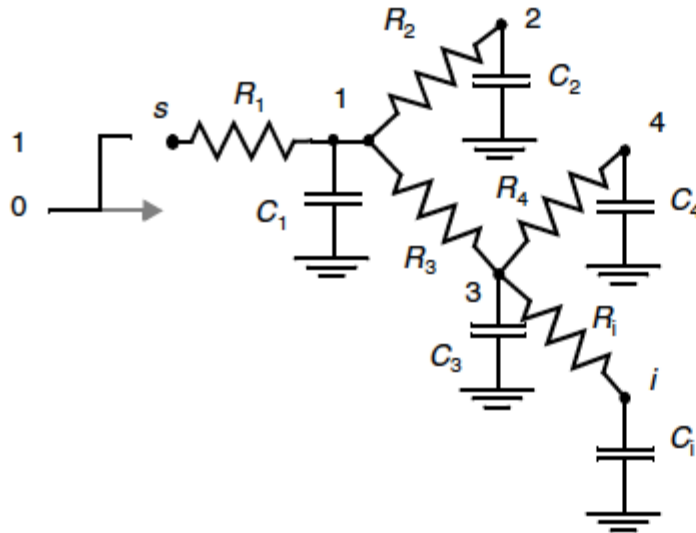


**Figure 4.11** Distributed versus lumped capacitance model of wire.  $C_{lumped} = L \times c_{wire}$ , with  $L$  the length of the wire and  $c_{wire}$  the capacitance per unit length. The driver is modeled as a voltage source and a source resistance  $R_{driver}$ .

# Elmore Delay Formula



- Assumptions regarding the RC network
  - the network has a single input node
  - all the capacitors are between a node and the ground
  - the network does not contain any resistive loops (tree)
- Unique resistive path
  - path resistance  $R_{44} = R_1 + R_3 + R_4$
  - shared path resistance  $R_{ik} = \sum R_j \Rightarrow (R_j \in [path(s \rightarrow i) \cap path(s \rightarrow k)])$



$$\tau_{Di} = \sum_{k=1}^N C_k R_{ik}$$

Figure 4.12 Tree-structured RC network.



Questions?

Comments?

Discussion?



## Acknowledgement

N. Weste and D. Harris, “CMOS VLSI Design”, 2011

Jan Rabaey, “Digital Integrated Circuits”, 2006

Cornell University, ECE 5745

UC Berkeley, CS 230

MIT, 6.371